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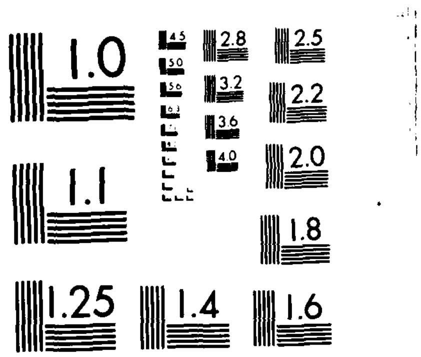


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# Field Validation of Statistically Based Acceptance Plan for Bituminous Airport Pavements



Volume 4—Computer Simulation of
Multiple Acceptance Criteria

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16. Abstract
This report presents the procedures and results of a computer simulation analysis conducted to investigate the performance of 7 methods for determining the payment factor for a lot of materials when 3 correlated acceptance properties, i.e., the Marshall stability, flow and air voids, are used for acceptance purposes. The methods investigated were based upon triple numerical integration of the 3 properties, multiplying the individual property values, averaging the individual property values and using the smallest individual property value.

Marshall test results from 15 runway paving projects were analyzed to determine mean, variance and correlation values obtained in field construction. Computer simulation was used to investigate the performance of the various methods for determining the payment factor for multiple acceptance properties. Simulation was used to generate Marshall test results for using population values from the 15 field projects. The mean square payment error (MSE) for each method was used as the norm and the minimum MSE as the criterion for choice among the methods.

It is recommended that the average of the 3 payment factors for the individual Marshall properties be determined using the quality index approach currently employed by the FAA Eastern Region for density acceptance purposes. The payment factor for the Marshall properties can then be calculated as the average of the 3 individual property payment factors.

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## PREFACE

This report presents the findings of a research project entitled "Field Validation of Statistically-Based Acceptance Plan for Bituminous Airport Pavements", Report No. DOT/FAA/PM-84/12, that was conducted to investigate the use of Marshall properties for acceptance purposes. The results of the research effort are presented in the series of reports listed below:

Burati, J.L., Brantley, G.D. and Morgan, F.W., "Correlation Analysis of Marshall Properties of Laboratory-Compacted Specimens," Final Report, Volume 1, Federal Aviation Administration, May, 1984.

Burati, J.L., Seward, J.D. and Busching, H.W., "Statistical Analysis of Marshall Properties of Plant-Produced Bituminous Materials," Final Report, Volume 2, Federal Aviation Administration, May, 1984.

Burati, J.L. and Seward, J.D., "Statistical Analysis of Three Methods for Determining Maximum Specific Gravity of Bituminous Concrete Mixtures," Final Report, Volume 3, Federal Aviation Administration, May, 1984.

Nnaji, S., Burati, J.L. and Tarakji, M.G., "Computer Simulation of Multiple Acceptance Criteria," Final Report, Volume 4, Federal Aviation Administration, August, 1984.

Burati, J.L., Busching, H.W. and Nnaji, S., "Field Validation of Statistically-Based Acceptance Plan for Bituminous Airport Pavements -- Summary of Validation Studies," Final Report, Volume 5, Federal Aviation Administration, September, 1984.

The application of multiple price adjustments is significantly more involved than the case when only one property, e.g., density, is considered. Since the Marshall properties (i.e., stability, flow and air voids) are physically related, they can be expected to be statistically correlated. If this is truly the case, then it may not be sufficient to treat each of the three properties individually. It is necessary to determine whether correlations exist among these properties, and whether such correlations should be considered when developing acceptance plans.

The objectives of the research described in the reports listed above include:

1. Review current methods for determining maximum specific gravity for use in air voids calculations for possible incorporation into the FAA Eastern Region P-401 specification,

2. Investigate the use of price adjustments when more than one characteristic is being used for acceptance purposes and recommend to the FAA potential procedures for dealing with multiple price adjustments,

3. Develop the procedures necessary to evaluate the performance of multiple properties acceptance plans,

4. Implement proposed Marshall properties acceptance plans on demonstration projects under field conditions, and

5. Attempt to correlate values of asphalt content and aggregate gradation with those from Marshall tests to determine whether or not correlations exist among these properties.

This report, Volume 4, presents the results of computer simulation analyses used in the development and evaluation of multiple-property price adjustment systems. The results of laboratory analyses and an analysis of field data for the correlation among the Marshall properties are presented in Volumes 1-3.

# CHAPTER I

# INTRODUCTION

In the early 1960's, a new philosophy with respect to the bases, language, understanding, and enforcement of the quality control of asphalt pavements evolved. The new philosophy recognized 3 previously neglected aspects of quality control:

1. specifications should account for the inherent variability of the materials and the testing procedures,

2. specifications should be more concerned with the end result rather than the resources used and the procedures followed, and

3. contract responsibilities and authority should be divided fairly between the owner (and/or his representative at the site) and the contractor.

State highway departments began incorporating these concepts into specifications in the form of end-result statistically-based specifications as early as the late 1960's. The Federal Aviation Administration (FAA) Eastern Region, for the first time incorporated statistically-based acceptance approaches in its bituminous surface course specifications (Item P-401) during the 1978 paving season.

In 1980, the FAA Eastern Region incorporated a statistically-based price adjustment schedule for the mat density of asphalt pavements based on an FAA-sponsored research project (1). However, the final report of that research project did not offer a multiple price adjustment system for all the acceptance variables (Marshall stability, flow, and air voids, and mat density) that the FAA desired. The final report did not recommend a multiple price adjustment system for all the acceptance variables because these variables are known to be physically related and further study was needed to quantify and analyze these relationships before recommending such a price adjustment system. Correlatons are important in developing a price adjustment schedule because they measure the variation of one variable relative to that of another. Hence, if the correlation between 2 variables is ignored and the 2 variables are treated independently, the inter-relationship of the 2 variables will not be taken into account, possibly resulting in partially measuring the same material characteristic twice.

## Scope

The term 'asphalt pavements' refers to many different types of paving courses designed for different uses and based on different properties of the mixture and the components. It is used in this report to denote plant-produced, hot-mixed, hot-laid, dense-graded bituminous

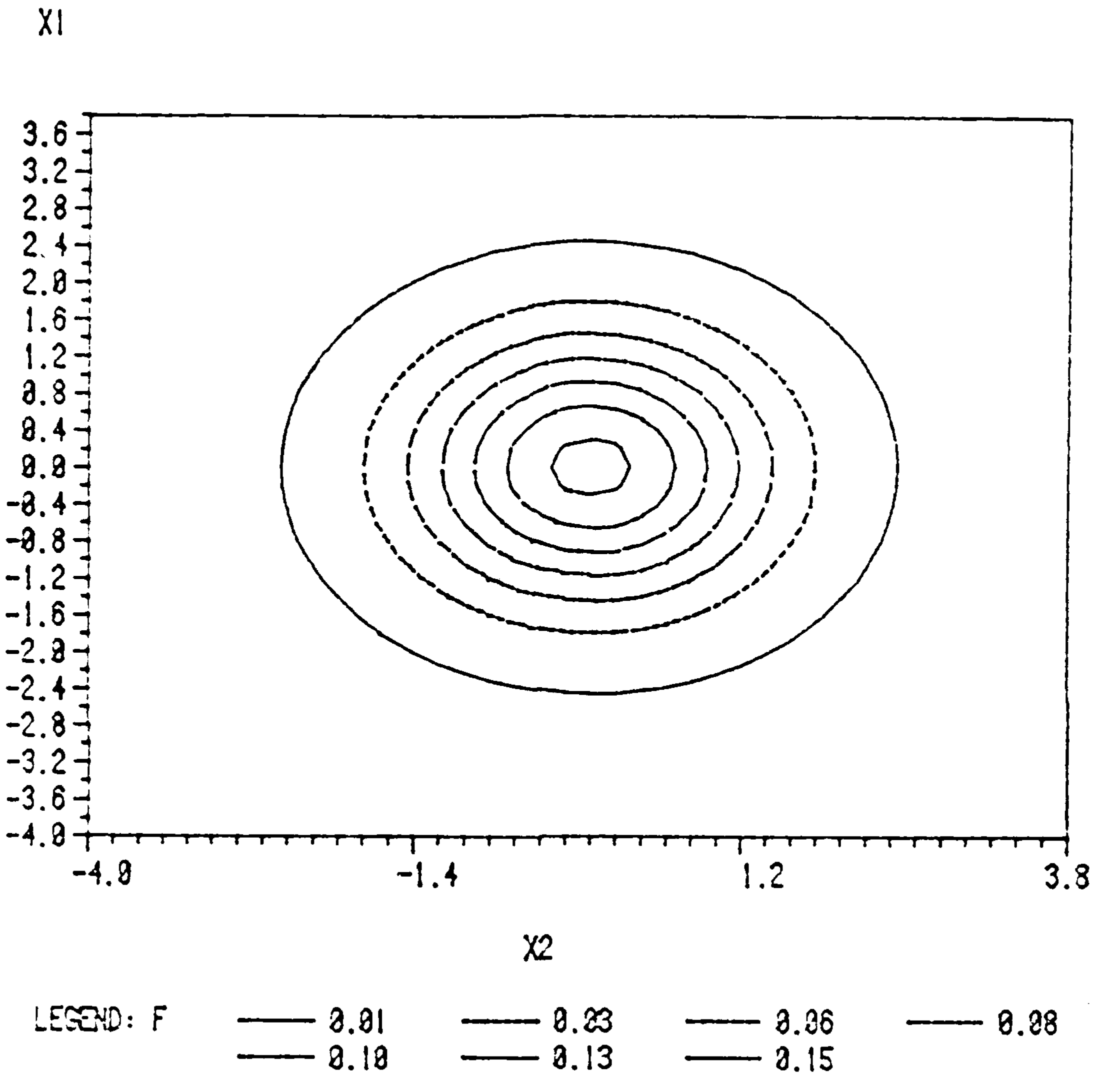


Figure 6. Contours of the Bivariate Normal Distribution, 0.0 Correlation.

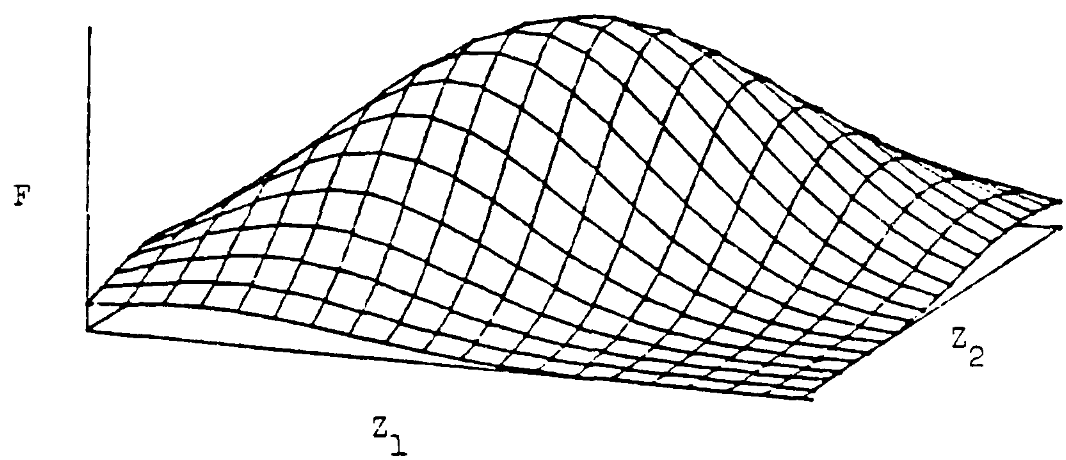


Figure 5. Shape of the Bivariate Normal Distribution, -0.6 Correlation.

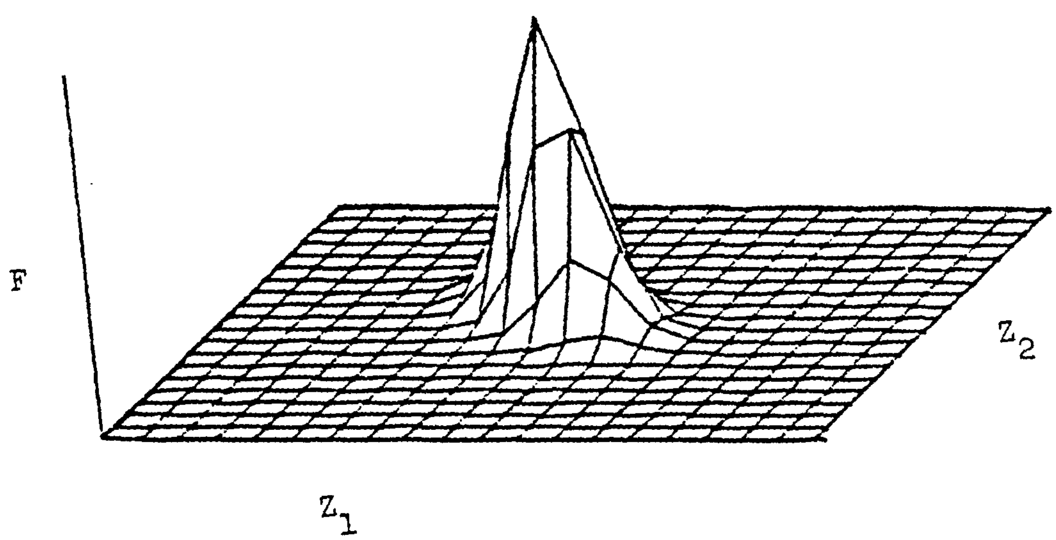


Figure 4. Shape of the Bivariate Normal Distribution, +0.6 Correlation.

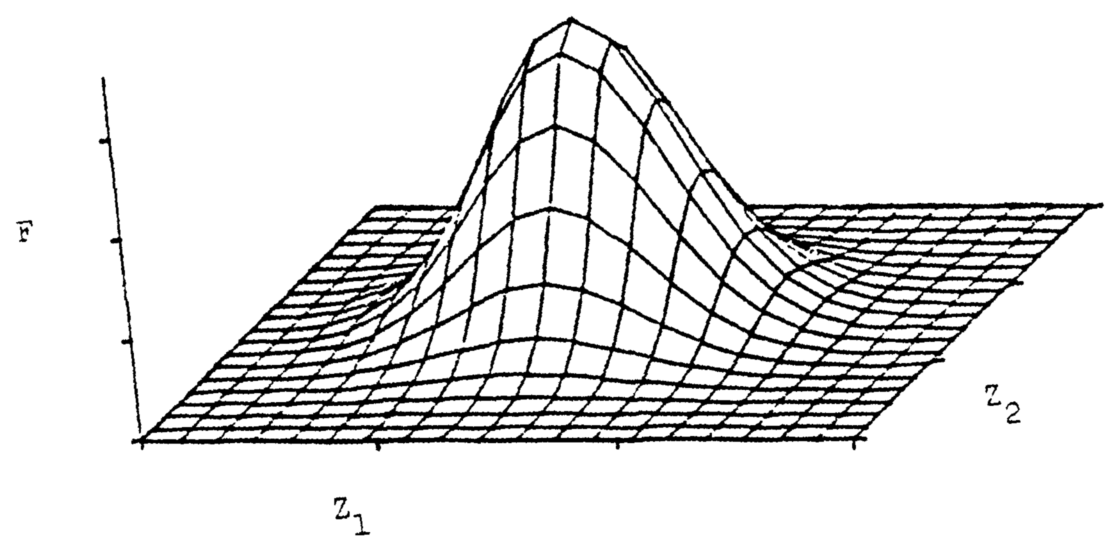


Figure 3. Shape of the Bivariate Normal Distribution, 0.0 Correlation.

For the bivariate case, the pdf is:

$$f(z) = \frac{1}{(2\pi)\,|C|^{\frac{1}{2}}} \{\exp \; -0.5 \begin{bmatrix} Z1 \\ Z2 \end{bmatrix} \begin{bmatrix} C11 & C12 \\ C21 & C22 \end{bmatrix} [Z1 \;\; Z2] \}$$

where  C11 = C22 = 1.0,
C12 = C21 = correlation between the 2 variables,

which can be simplified to (11):

$$f(z) = \frac{1}{(2\pi) \quad (1-P^2)^{\frac{1}{2}}} \exp(Gz)$$

where  Gz = 0.5 [1/{1-(P*P)}]*[(Z1*Z1) - 2*P*(Z1*Z2) + (Z2*Z2)],

P = C12 = correlation between Z1 and Z2.

Figures 3, 4, and 5 show the shape of the bivariate normal distribution with zero correlation, positive correlation (P = 0.6), and negative correlation (P = -0.6), respectively. The shape of the bivariate normal distribution is relevant to the volume beneath it (PWL). For example, the PWL under the bivariate normal curve between minus infinity and zero for both variables is 0.25 for zero correlation, 0.35 for +0.6 correlation, and 0.1[illegible]3 for -0.6 correlaton. Figures 6, 7, and 8 show contours for the bivariate pdf's. The shape of the trivariate normal distribution can not be shown because it is a 4 dimensional volume in Euclidean space.

When Cxy has a numeric value of 0.0 the 2 variables are said to be uncorrelated and can, therefore, be treated separately. When Cxy has a numeric value of 1.0 or -1.0 the 2 variables are said to have perfect correlation. That is, all the variation in one can be predicted exactly by the variation of the other.

## Multivariate Normal Distributions

The multivariate normal distribution for n variables with a mean vector M and covariance matrix V (non-singular) is defined by the following probability density function (pdf):

$$f(X) = (2\pi)^{-k/2} \mid V \mid^{-1/2} \exp \{-0.5([X-M]^T [V]^{-1} [X-M])\}$$

where
$[V]$ = matrix or vector V,
$|V|$ = determinant of V,
X = correlated normal variates vector,
M = means vector,
T = transposed matrix operation
k = number of variables.

For the standardized multivariate normal distribution having means equal to zero and variances equal to 1.0, the pdf becomes:

$$f(x) = (2\pi)^{-k/2} \mid C \mid^{-1/2} \exp \{-0.5([Z]^T [C]^{-1} [Z])\}$$

where
Z = standardized variates vector,
C = correlation matrix (matrix of correlation coefficients of the variables).

For the trivariate case, the pdf is:

$$f(z) = \frac{1}{(2\pi)^{3/2} \mid C \mid^{1/2}} \exp \{-0.5([Z]^T[C]^{-1}[Z])\}$$

where $[Z] = [\ Z1 \quad Z2 \quad Z3\ ]$,

$$[C] = \begin{bmatrix} C11 & C12 & C13 \\ C21 & C22 & C23 \\ C31 & C32 & C33 \end{bmatrix},$$

C11 = C22 = C33 = 1.0,
C12 = C21 = correlation between variates 1 and 2,
C13 = C31 = correlation between variates 1 and 3,
C23 = C32 = correlation between variates 2 and 3.

# CHAPTER II

## STATISTICAL BASIS

In the process of developing an acceptance plan for the Marshall properties, it is important to base the plan on the results of a large number of data points. Because it is infeasible to actually perform the large number of Marshall tests required, it is important to be able to generate hypothetical test results. One way to generate such results is by computer simulation. The computer simulation process (Chapter III) requires a random number generator routine, a random normal number converter routine, and a matrix decomposition algorithm. Previous research (4) has shown that it is acceptable to assume that the 3 Marshall properties (stability, flow, and air voids) have individual normal distributions. Since these properties vary simultaneously, and are therefore physically related, it is reasonable to assume that the 3 together have a trivariate normal distribution.

It is shown in Chapter I that the PWL of a function with 2 or more variables is a volume in Euclidean space. Thus, to calculate the PWL for the Marshall properties, it is necessary to integrate the trivariate normal probability density function (pdf). This integral, the trivariate normal cumulative distribution function (CDF), can not be obtained analytically. Consequently, the only practical way to determine the trivariate PWL is by numerical integration.

### Correlation Coefficient

The correlation coefficient is a measure of the linear association between variables. The mathematical equation for the correlation between 2 variables X and Y, Cxy, is given by:

$$Cxy = \frac{\Sigma\ (Xi-\bar{x}) * (Yi-\bar{y})}{[\ \Sigma\ (Xi-\bar{x})^2 * \Sigma\ (Yi-\bar{y})^2\ ]^{1/2}}$$

where Cxy is the correlation coefficient
X1, X2 ... Xi ... Xn are the observations for X,
Y1, Y2 ... Yi ... Yn are the observations for Y,

$\bar{x}$ is the mean of X,

$\bar{y}$ is the mean of Y.

Table 4. Experimental Design and Number of Replicates Used in Marshall Laboratory Analysis (3)

| Gradation | Asphalt Content (%) | | | | | |
|---|---|---|---|---|---|---|
| | 5.0 | 5.5 | 6.0 | 6.5 | 7.0 | 7.5 |
| A | 12 | 12 | 12 | 12 | 12 | 12 |
| B | 12 | 12 | 12 | 12 | 12 | 12 |
| C | 12 | 12 | 12 | 12 | 12 | 12 |
| D | 12 | 12 | 12 | 12 | 12 | 12 |

Table 2. 1978 Paving Projects On Which Data Were Collected (1)

| Project | Location |
|---|---|
| Adirondack-A* | Saranac Lake, NY |
| Adirondack-B* | Saranac Lake, NY |
| Charlottesville-ANJ* | Charlottesville, VA |
| Charlottesville-SLW* | Charlottesville, VA |
| Chautauqua | Jamestown, NY |
| Chemung-Chemung* | Elmira, NY |
| Chemung-Fisherville* | Elmira, NY |
| DuBois | DuBois, PA |
| Dutchess | Poughpkeepsie, NY |
| Linden | Linden, NJ |
| Westchester-Colprovia* | White Plains, NY |
| Westchester-Peckham* | White Plains, NY |

*On these projects two asphalt plants, each with a different JMF, were used

Table 3. 1981 Paving Projects On Which Data Were Collected (2)

| Project | Location |
|---|---|
| Baltimore Washington International (BWI) | Baltimore, MD |
| National Aviation Facilities Experimental Center (NAFEC), Atlantic City | Pamona, NJ |
| Monroe County Airport (Rochester) | Rochester, NY |

## Sources of Data

A total of 288 laboratory samples along with data from 15 runway paving projects were used in this study. The data used are from the following sources:

1. Field data, collected by Burati and Willenbrock (1), from 12 paving projects provided by the FAA Eastern Region during the 1978 paving season. The projects and their locations are listed in Table 2.

2. Field data, collected by Burati and Seward (2), from 3 paving projects provided by the FAA Eastern Region during the 1981 construction season. These projects and their locations are listed in Table 3.

3. Laboratory data from 24 bituminous concrete mixtures, 12 replicates per mixture, prepared and tested by Brantley (3). The experimental design for these data is shown in Table 4.

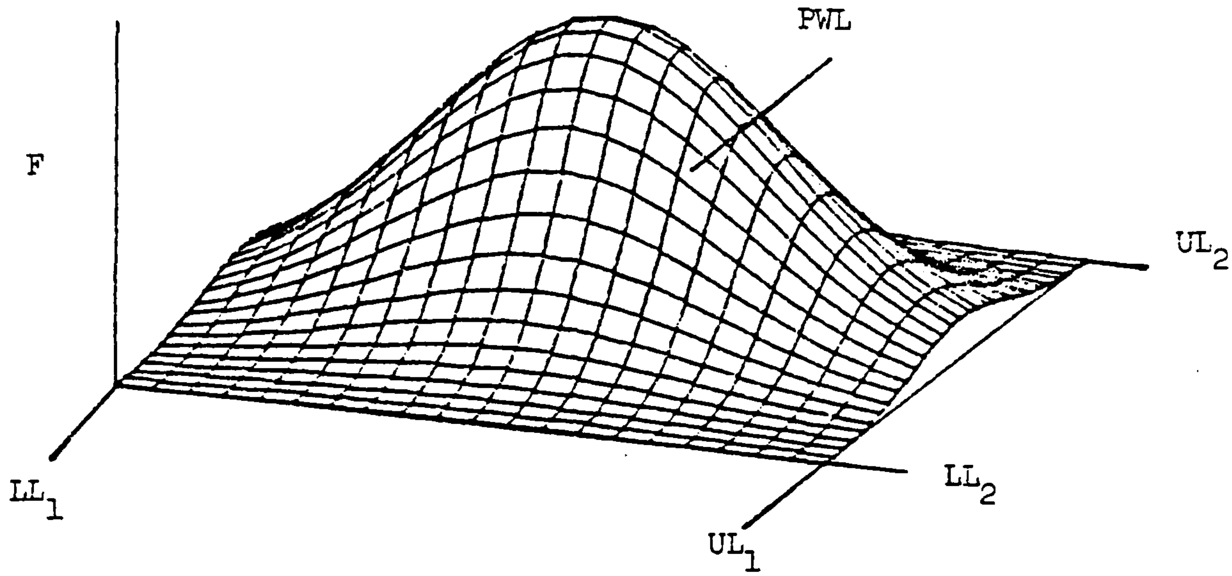


Figure 2. Illustration of PWL for Two Variables with Upper (UL1 and UL2) and Lower (LL1 and LL2) Acceptance Limits.

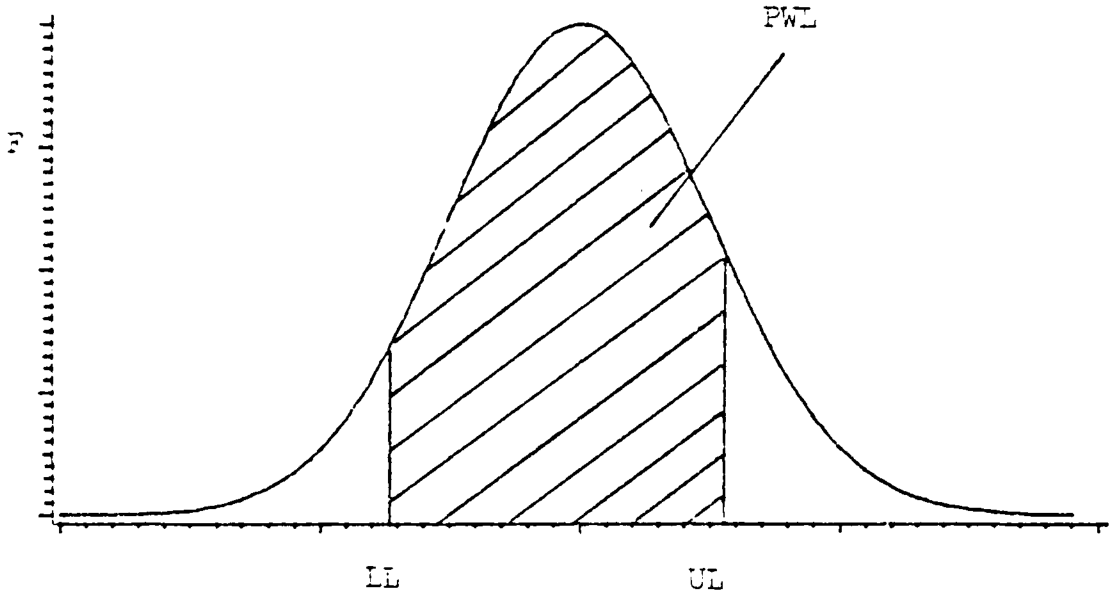


Figure 1. Illustration of PWL for a Single Variable with Upper (UL) and Lower (LL) Acceptance Limits.

Table 1. FAA Marshall and Mat Density Acceptance Criteria (Heavy Construction)

| Test Item[a] | Lower Limit | Upper Limit |
|---|---|---|
| Stability (pounds) | 1800 | none |
| Flow (0.01 inches) | 8 | 16 |
| Air Voids (%) | 2 | 5 |
| Mat Density[b] (%) | 96.7 | none |

a. For substantial compliance, 90% or more of the material should be within limits.

b. For the mat density, payment is adjusted according to Table 18 for materials with less than 90 PWL.

concrete prepared according to the FAA Eastern Region's specifications to be used in surfacing airport runways, taxiways and aprons.

The FAA specifications use the Marshall stability, flow and air voids as acceptance criteria for plant mixtures; and the pavement density, thickness, smoothness and grade for the construction workmanship. Furthermore, the FAA requires the contractor to correct any deficiencies in the pavement's thickness and surface conditions (P-401). Consequently, these items are not included in this study because they are treated by the FAA as correctable, and are not considered for price adjustments. Table 1 presents the FAA acceptance limits for Marshall stability, flow and air voids and for mat density.

## Research Objectives

The goal of this study is to formulate a procedure for the acceptance and the application of price adjustment factors to asphalt airport pavements using the acceptance variables currently used by the FAA Eastern Region. This goal has the following specific objectives:

1. Study the correlation structure between the 3 Marshall properties (stability, flow and air voids) and their effect on the acceptance criteria of asphalt pavements.

2. Develop an algorithm to measure the quality of the pavement as a function of the 3 Marshall properties.

3. Investigate the adequacy of present heuristic methods for evaluating compliance using more than one acceptance property.

## The PWL Concept

The percentage within limits (PWL) is a concept used in statistically-based acceptance plans to incorporate both the mean and the variability of the test results in determining the acceptability of the lot. The PWL represents the percentage of the lot falling within the acceptance limits. Thus, for the 2 dimensional case of a single variable such as density, the PWL is the area under the density distribution function of the variable between the upper and the lower limits of acceptance specified for that variable (Figure 1). For the 3 dimensional case of 2 variables, the PWL is the volume under the bivariate distribution surface between the 4 acceptance limits (one upper and one lower limit for each variable). Figure 2 shows the volume representing the PWL of 2 variables. For more than 2 variables, the PWL is a volume in Euclidean space bounded by the upper and lower limits for each variable.

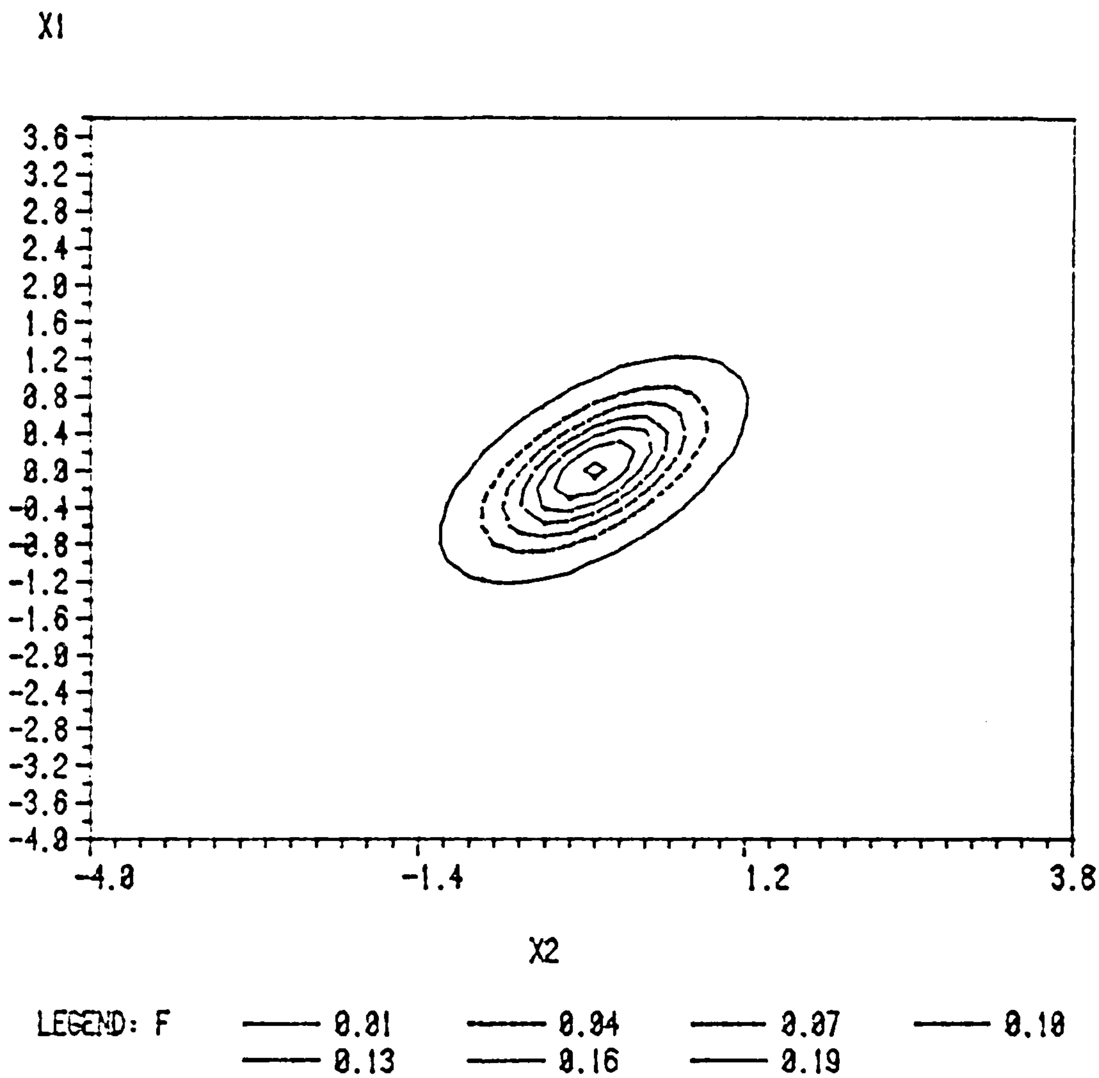


Figure 7. Contours of the Bivariate Normal Distribution, +0.6 Correlation

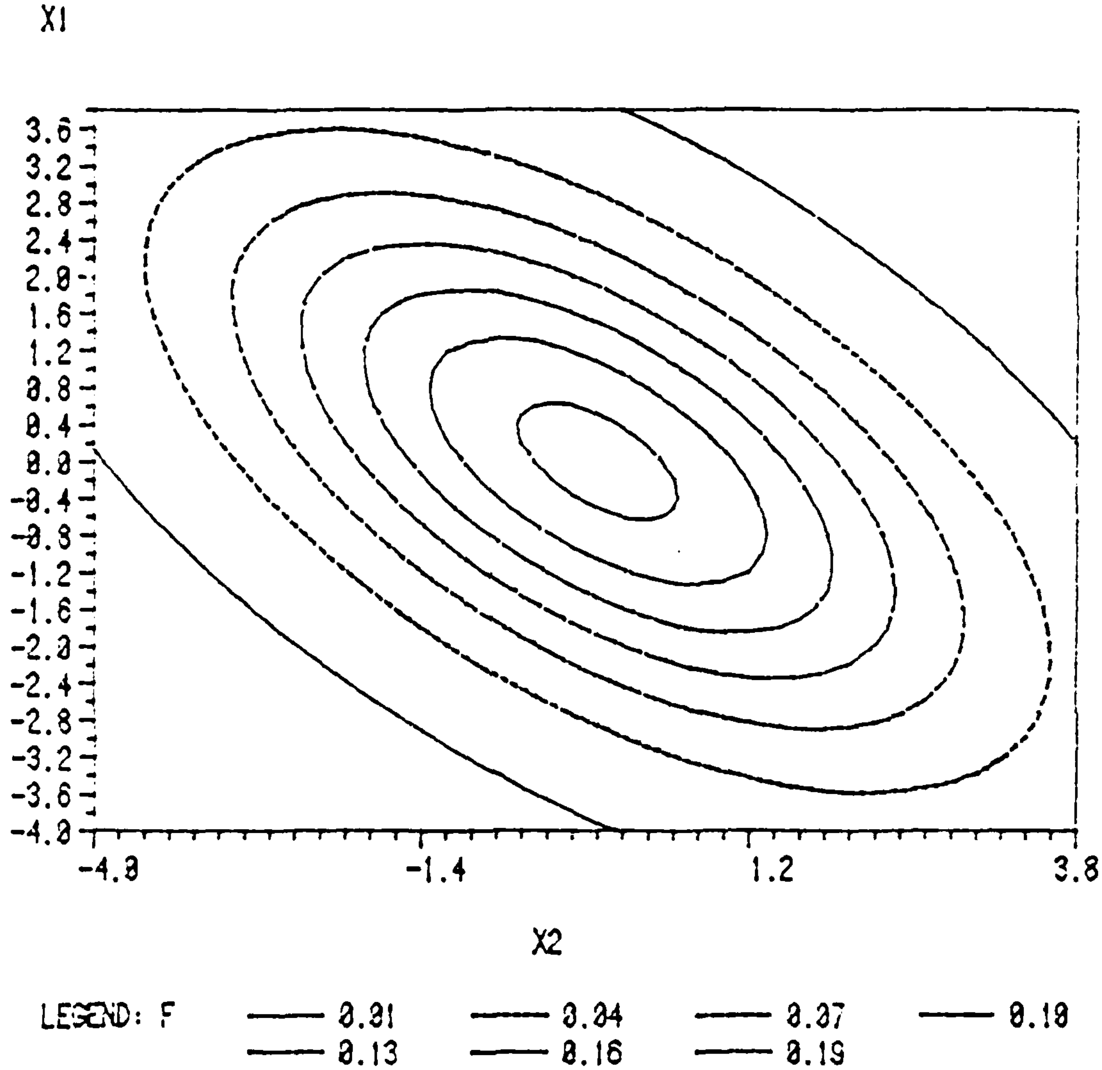


Figure 8. Contours of the Bivariate Normal Distribution, -0.6 Correlation

# CHAPTER III

## NUMERICAL COMPUTATIONAL PROCEDURES

In this chapter, the numerical computational procedures used in developing the multivariate acceptance plan are described. These include:

1. multivariate computer simulation to generate hypothetical Marshall test results, and

2. numerical integration to obtain a measure of the quality of the asphalt mixture in terms of the trivariate PWL.

### Multivariate Computer Simulation

Random normal variates can be generated using different techniques. The central limit theorem, the Box-Muller method and the Morsaglia polar method (5) are some of these techniques. The Morsaglia polar method, which is a modification of the Box-Muller method, was selected for use in this study due to its superiority in a preliminary evaluation. Table 5 shows the performance of function RNOR (6), which is based on the Morsaglia polar method, and subroutine GAUSS from IBM's Scientific and Statistical Package (7), which is based on the central limit theorem, for 500 simulations.

The simulation of correlated normal variates can be accomplished by the following matrix operations: let [Z] be a vector of random normal variates of size m, [V] a vector of means of size m, [C] a covariance matrix of size m*m, and [R] a lower triangular matrix such that $[R] * [R]^T = [C]$.
Then,

$$[X] = [V] + ([Z] * [R])$$

is a vector of correlated random normal variates (8). The Cholesky Sequential Decomposition Algorithm, CSDA, converts a matrix, [C], to a lower triangular matrix, [R]. Subroutine CSDA was tested by simulating 1000 Marshall tests using the statistics of each of the 1978 projects as population values. The 3 mean simulated correlation coefficients for each project (a total of 36 correlation coefficients for the 12 projects), were calculated and compared with the population correlations. As shown in Figure 9, the maximum difference between the population correlations and the simulated correlatons is less than 0.10, and the majority of these differences range between 0.00 and 0.06. These small differences indicate that subroutne CSDA is an acceptable generator of correlated random normal numbers.

Table 5. Performance of the Central Limit Theorem and the Morsaglia Polar Method for Generating 500 Pseudo-Normal Random Numbers

| | Central Limit Limit Theorem | Morsaglia Polar Method |
|---|---|---|
| Average Error in Mean | 0.0444 | 0.0149 |
| Average Error in Standard Deviation | 0.0236 | 0.0004 |
| Computer Execution Time (seconds) | 0.41 | 0.09 |

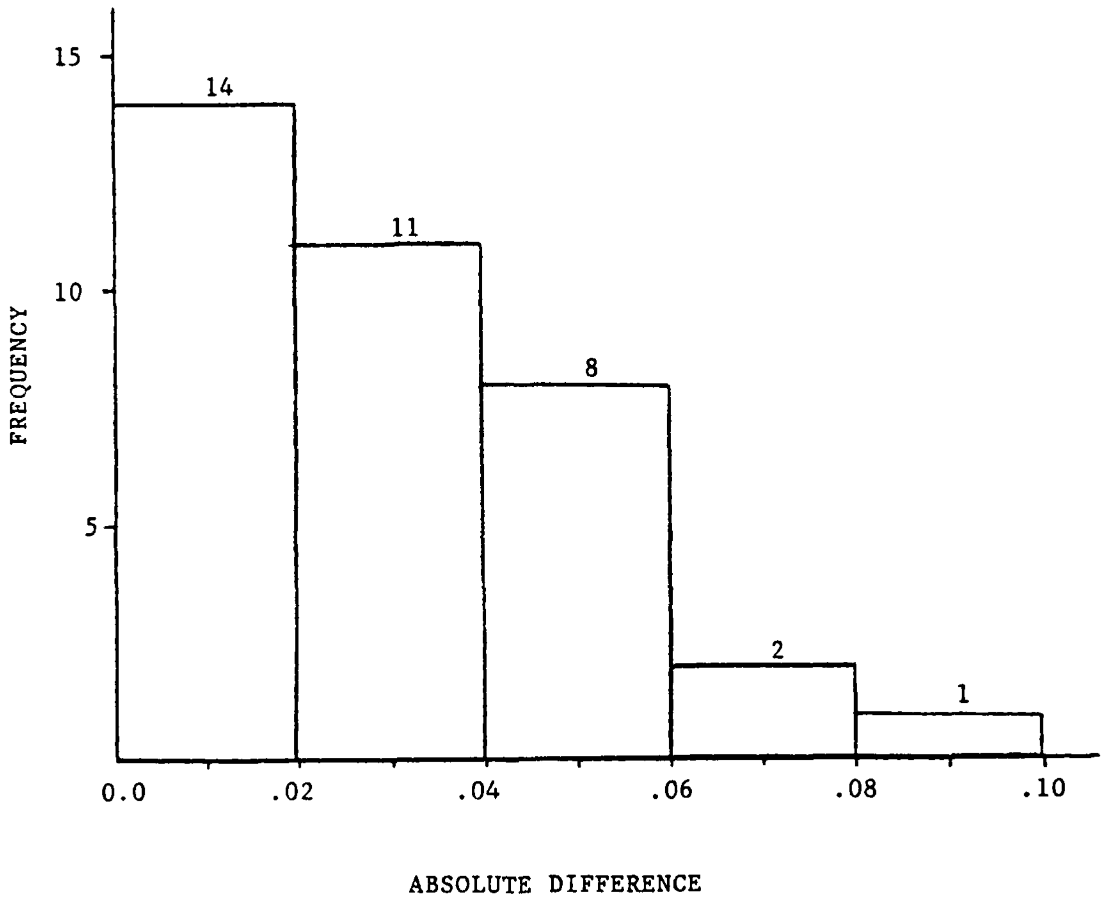


Figure 9. Histogram of the Differences Between the Simulated Correlations and the True Correlations Between the Marshall Properties of the 1978 Paving Projects Using Subroutine CSDA.

## Numerical Integration

The integral of a continuous non-negative function f(x), with a closed interval domain (a,b), is the area of the region under the curve betweeen the 2 vertical lines x=a and x=b. For example, if f(x)=x, then $\int f(x)\, d(x)$ is the area under the 45 degree line between x=a and x=b.

For functions with more than one variable, Davis (9) recommends the use of the Cartesian products and product rule to obtain the volume in Euclidean space that corresponds to the double or higher level integral. To illustrate this approach using vector notation, let B be a region with points $X=(x_1, x_2 \ldots x_n)$ and G be a region with points $Y=(y_1, y_2 \ldots\ldots y_n)$, then

$$\int f(x)\, dv = (1/m) \Sigma\ w_i\ f(x_i),$$

$$\int f(y)\, dv = (1/m) \Sigma\ v_j\ f(y_j),$$

$$\text{Volume} = \int f(x,y)\, dv = (1/m) \Sigma\Sigma\ w_i\ v_j\ f(x_i, y_j).$$

where $w_i$ and $v_j$ are the weights from the compound Simpson's rule,

m is the total number of weights

$f(x_i, y_j)$ is the value of the function at $X=x_i$ and $Y=y_j$.

The weighted values can be obtained using the following relation:

$$W = \Pi\ w(i).$$

Subroutine SUM, shown in Appendix D, calculates the weighted average for functions with any number of variables. This subroutine was adapted from Davis (9).

## Numerical Integration Boundaries

For the one degree normal distribution, 99.73 percent of the area under the curve lies within 3 standard deviations from the mean. When numerical integration is used, the boundary limits should be carefully selected. If they are too close to the mean, significant areas in the tails will be lost. If they are too far from the mean, the function will be evaluated at too many points in the extreme portions of the tails where the function value is practically zero. This results in underestimating the weighted average of the function. The area depends on both the integration interval and the weighted average value of the function.

To select boundary limits for the multivariate normal distribution, the volume in Euclidean space under the trivariate normal distribution for the zero correlations case was calculated for various boundary limits (Table 6). The boundary limit standing for infinity was selected to be 3.5 standard deviations because the volume calculated by numerical integration for variate intervals [-3.5, +3.5] was 0.9949 which is very

close to the theoretical value (the product of the 3 individual areas as obtained from normal distribution tables), 0.9988, and close to the total volume under the surface which is 1.0. The same conclusion was made when the upper limits were taken as zeros. Selection of -3.5 as a lower limit standing for minus infinity gave a volume of 0.1248. This is close to both the theoretical value (0.1249) and the total volume of 0.125 for the region from minus inifinity to zero.

## Selection of the Integration Rule

The 17-point integration rule (evaluating the function at a grid of 17 points in each dimension) was selected for 2-degree numerical integration. The 9-point integration rule was selected for 3-degree numerical integration. It was found that accurate results could be obtained at these levels. Using more points does not significantly increase the accuracy of the results, but does dramatically increase computer execution time. Tables 7 and 8 show the integration results for the standardized bivariate normal distribution and the standardized trivariate normal distribution for variate intervals [-3.0, +3.0] and zero correlations.

Because the variates are uncorrelated, the theoretical values corresponding to the integration intervals used can be calculated from a normal distribution table (Appendix A) by multiplying the corresponding variate areas. Thus, for variate intervals [-3.0, +3.0], the volume under the bivariate normal surface is 0.9974 * 0.9974 = 0.9948, and the volume under the trivariate normal surface is 0.9974 * 0.9974 * 0.9974 = 0.9922.

The fact that a higher integration rule was selected for the 2-degree case than for the 3-degree case can be explained by the fact that the bivariate 17-point rule evaluates the integral at 289 (17 * 17) points, while the trivariate 9-point rule evaluates the integral at 729 (9 * 9 * 9) points.

Table 6. Volumes Under the Trivariate Normal Distribution for the Zero Correlations Case Using Different Integration Intervals

| Lower Boundary Limits | Upper Boundary Limits | Numerical Integration Volume | Theoretical[a] Volume (Tables) |
|---|---|---|---|
| -3.5 | 3.5 | 0.9949 | 0.9988 |
| -3.0 | 3.0 | 0.9908 | 0.9922 |
| -3.5 | 0.0 | 0.1248 | 0.1249 |
| -3.0 | 0.0 | 0.1240 | 0.1240 |

[a] The total Volume under the surface from minus infinity to plus infinity for all three variables is 1.0, and the total Volume under the surface from minus infinity to zero for all three variables is 0.125.

Table 7. Numerical Integration Results for the Bivariate Normal Distribution for Standardized Variate Intervals [-3.0, +3.0] and Zero Correlation

| Integration Points Rule | Number of Points of Function Evaluation | Volume[a] |
|---|---|---|
| 3 | 9 | 2.57615 |
| 5 | 25 | 1.62293 |
| 9 | 81 | 1.22030 |
| 17 | 289 | 0.99506 |
| 33 | 1,089 | 0.99509 |

[a]Theoretical Volume = 0.9948

Table 8. Numerical Integration Results for the Trivariate Normal Distribtuion for Standardized Variate Intervals [-3.0, +3.0] and Zero Correlations

| Integration Points Rule | Number of Points of Function Evaluation | Volume[a] |
|---|---|---|
| 5 | 125 | 2.06752 |
| 9 | 729 | 0.99161 |
| 17 | 4,913 | 0.99203 |

[a]Theoretical Volume = 0.9922

# CHAPTER IV

# DATA ANALYSIS

Marshall data from the 1978 and 1981 paving projects were analyzed to calculate the means, standard deviations, correlation coefficients, and the trivariate PWLs (the 3-degree numerical integration of the trivariate normal distribution using the FAA acceptance values for stability, flow, and air voids as the integration interval boundaries) for each project. Tables 9-12 present the means, standard deviations, correlation coeffficients, and PWL values for the 1978 projects. Tables 13-16 present the statistics for the 1981 paving projects.

Laboratory data obtained by Brantley (4) were also used to study the correlation coefficients and their variation. Brantley investigated the variation of the Marshall correlations as a function of asphalt content and aggregate gradation. Table 17 lists the correlation coefficients for the laboratory data for stability and flow, stability and air voids, and flow and air voids.

## General Trends

The standard deviations from the 1981 paving projects are generally lower than those for the 1978 paving projects. For the 1981 proejcts, the pooled standard deviations for stability, flow, and air voids are 244, 0.84, and 0.62, respectively. For the 1978 projects, the corresponding standard deviations are 279, 1.81 and 0.75, respectively. However, the values for 1981 are lower due to the results from the Baltimore-Washington project. The standard deviations for this project, particularly for stability and air voids, are quite low, and may not be indicative of typical projects. However, the fact that the magnitudes of the standard deviations are low does not necessarily indicate that the correlations among the results are not appropriate. For this reason, it was decided to include this project among those analyzed in the simulation analyses.

In both the field and laboratory data, there is a strong tendency for the correlation between stability and flow, C(s,f), to be positive, for the correlation between stability and air voids, C(s,v), to be negative, and for the correlation between flow and air voids, C(f,v), also to be negative.

For the 1981 projects, C(s,f) had 2 positive values and 1 negative value, C(s,v) had 3 negative values and no positive values, and C(f,v) had 2 negative values and 1 positive value (Table 15). For the 1978 projects, C(s,f) had 9 positive values and 3 negative values, C(s,v) had 10 negative values and 2 positive values, and C(f,v) had 11 negative values and 1 positive value (Table 11). These results are consistent with the laboratory correlatons which have 19 positive values and 5

negative values for C(s,f), 21 negative values and 3 positive values for C(s,v), and 20 negative values and 4 positive values for C(f,v) (Table 17).

To calculate representative correlations for the field data, the correlation values from both sources of field data for which the sign (positive or negative) did not agree with the general trend were deleted and the averages of those remaining were calculated. For C(s,f), there were 11 positive values with a mean of 0.278 which was rounded to 0.30. For C(s,v), there were 13 negative values with a mean of -0.396 which was rounded to -.40. For C(f,v), there were 13 negative values with a mean of -0.405 which was rounded to -0.40. The term 'trend correlations' is used to refer to the following values for the correlation coefficients between stability and flow, stability and air voids, and flow and air voids, respectively: +0.30, -0.40, and -0.40.

Table 9. Means of the Marshall Results for the 1978 Field Data

| Project | Stability | Flow | Voids |
|---|---|---|---|
| Adirondack-A | 2240.1 | [illegible] | 3.43 |
| Adirondack-B | 2341.6 | 10.[illegible] | 3.58 |
| Charlottesville-ANJ | 2702.6 | 15.91 | 2.68 |
| Charlottesville-SLW | 3614.7 | 15.27 | 3.62 |
| Chautauqua | 2450.4 | 9.59 | 3.08 |
| Chemung-Chemung | 2427.3 | 10.39 | 3.46 |
| Chemung-Fisherville | 2475.9 | 9.05 | 3.68 |
| DuBois | 2056.1 | 10.44 | 3.58 |
| Dutchess | 2853.6 | 13.40 | 4.41 |
| Linden | 2117.9 | 11.90 | 3.86 |
| Westchester-Colprovia | 2816.1 | 11.41 | 3.69 |
| Westchester-Peckham | 2686.2 | 11.86 | 3.64 |
| pooled | 2410 | 11.87 | 3.55 |

Table 10. Standard Deviations of the Marshall Results for the 1978 Field Data

| Project | Stability | Flow | Voids |
|---|---|---|---|
| Adirondack-A | 288.51 | 1.692 | 0.722 |
| Adirondack-B | 256.46 | 1.445 | 0.623 |
| Charlottesville-ANJ | 271.32 | 1.349 | 0.577 |
| Charlottesville-SLW | 367.34 | 2.800 | 0.964 |
| Chautauqua | 126.06 | 0.799 | 0.310 |
| Chemung-Chemung | 156.15 | 1.243 | 0.293 |
| Chemung-Fisherville | 260.94 | 0.902 | 0.473 |
| DuBois | 173.49 | 1.458 | 0.672 |
| Dutchess | 193.68 | 1.325 | 0.410 |
| Linden | 127.45 | 1.246 | 0.684 |
| Westchester-Colprovia | 203.69 | 1.790 | 0.614 |
| Westchester-Peckham | 436.86 | 2.540 | 1.179 |
| pooled | 279 | 1.81 | 0.75 |

Table 11. Correlation Coefficients of the Marshall Results for the 1978 Field Data

| Project | Stability and Flow | Stability and Voids | Flow and Voids |
|---|---|---|---|
| Adirondack-A | 0.342 | -0.773 | -0.741 |
| Adirondack-B | 0.589 | -0.609 | -0.630 |
| Charlottesville-ANJ | -0.241 | -0.022 | -0.029 |
| Charlottesville-SLW | -0.548 | 0.335 | -0.408 |
| Chautaugua | 0.324 | -0.507 | -0.354 |
| Chemung-Chemung | 0.441 | 0.014 | -0.481 |
| Chemung-Fisherville | 0.458 | -0.389 | -0.381 |
| DuBois | 0.330 | -0.271 | -0.811 |
| Dutchess | 0.047 | -0.431 | -0.143 |
| Linden | -0.062 | -0.102 | 0.044 |
| Westchester-Colprovia | 0.243 | -0.498 | -0.287 |
| Westchester-Peckham | 0.132 | -0.685 | -0.587 |

Table 12. PWLs of the Marshall Results for the 1978 Paving Projects

| Project | Number of Tests | Marshall PWL |
|---|---|---|
| Adirondack-A | 9 | 62.4 |
| Adirondack-B | 29 | 81.1 |
| Charlottesville-ANJ | 54 | 69.4 |
| Charlottesville-SLW | 53 | 50.1 |
| Chautauqua | 27 | 93.3 |
| Chemung-Chemung | 24 | 94.3 |
| Chemung-Fisherville | 56 | 84.6 |
| DuBois | 32 | 75.4 |
| Dutchess | 12 | 88.7 |
| Linden | 52 | 94.0 |
| Westchester-Colprovia | [illegible] | 93.0 |
| Westchester-Peckam | 85 | 56.2 |

Table 13. Means of the Marshall Results for the 1981 Field Data

| Project | Stability | Flow | Voids |
|---|---|---|---|
| Atlantic City | 2487.1 | 10.02 | 3.43 |
| Baltimore-Washington | 2794.0 | 10.60 | 3.40 |
| Rochester | 3207.3 | 12.50 | 3.69 |
| pooled | 2671.8 | 10.55 | 3.47 |

Table 14. Standard Deviation of the Marshall Results for the 1981 Field Data

| Project | Stability | Flow | Voids |
|---|---|---|---|
| Atlantic City | 288.44 | 0.801 | 0.737 |
| Baltimore-Washington | 68.53 | 0.543 | 0.204 |
| Rochester | 222.87 | 1.236 | 0.447 |
| pooled | 243.8 | 0.84 | 0.62 |

Table 21. Summary of Population PWL Values for the Projects Simulated

| Project | Correlation Coefficients | | | PWL (corr.) | PWL (no corr.) |
|---|---|---|---|---|---|
| | stab-flow | stab-voids | flow-voids | | |
| Adirondack-A | 0.34 | -0.77 | -0.74 | 62.4 | 80.8 |
| Adirondack-B | 0.59 | -0.61 | -0.63 | 81.1 | 88.7 |
| Charlottesville-ANJ | -0.24 | -0.02 | -0.03 | 46.2 | 46.3 |
| Charlottesville-SLW | -0.55 | 0.34 | -0.41 | 49.9 | 52.4 |
| Chautauqua | 0.32 | -0.51 | -0.35 | 96.6 | 97.3 |
| Chemung-Chem | 0.44 | 0.01 | -0.48 | 94.3 | 96.9 |
| Chemung-Fish | 0.46 | -0.39 | -0.38 | 84.6 | 87.0 |
| Dubois | 0.33 | -0.27 | -0.81 | 75.4 | 86.2 |
| Dutchess | 0.05 | -0.43 | -0.14 | 88.7 | 90.0 |
| Linden | -0.06 | -0.10 | 0.04 | 94.0 | 94.1 |
| Westchester-Colp | 0.24 | -0.50 | -0.29 | 93.0 | 94.6 |
| Westchester-Peck | 0.13 | -0.69 | -0.59 | 56.2 | 68.6 |
| Atlantic City | 0.07 | -0.33 | -0.30 | 92.8 | 94.2 |
| Baltimore-Wash. | -0.60 | -0.30 | 0.08 | 99.0 | 99.5 |
| Rochester | 0.09 | -0.24 | 0.01 | 99.3 | 99.3 |
| Trend * | 0.30 | -0.40 | -0.40 | 97.0 | 98.1 |

* On this project, pooled values for mean and standard deviation from the 1981 projects were used along with the trend correlations. The means used were 2672, 10.55 and 3.47 for stability, flow and air voids. The standard deviations were 243.8, 0.84 and 0.62.

The population PWL values are calculated in Box 2 using the triple numerical integration routine and the 'population' statistics input in Box 1. The algorithm used to effect the integration is adapted from an algorithm (P9) given by Davis (9) and is based on a 9-point integration rule (see Chapter 3). The limits of integration are the respective acceptance limits. These limits are standardized for the purposes of implementation. Thus, if XU and XL are the upper and lower limits for variable X, then the respective standardized limits are:

$$U = (XU - XBAR)/STD \quad \text{and} \quad L = (XL - XBAR)/STD$$

where XBAR and STD are the mean and standard deviation, respectively, input in Box 1.

The PWL value for the population is then used with the payment schedule in Table 18 to determine the correct payment for the lot. The simulation was conducted using each of the 15 project results as the 'population' statistics. The PWL values for these 15 populations appear in Table 21. To examine the effect of the correlation between variables on the PWL values, the PWL for the zero correlation case is also shown for each project in Table 21.

It can be seen in Table 21 that the effect of correlation is to reduce the PWL value from the zero correlation case. However, the effect is quite small for populations with high PWL values. For example, there is little decrease in PWL when correlation is considered for populations which would receive 100 percent payment, i.e., for populations with PWL values greater than 90. The large differences in PWL values occur in the table when either 1 or 2 large (greater than .60) negative correlation coefficients are present. For example, on the Adirondack-A project, where there are 2 large negative correlations (-0.77 and -0.74) there is an 18.4 point decrease in PWL when the correlations are considered. For Charlottesille-ANJ, on the other hand, which has very low correlaton coefficients, the PWL value only drops from 46.3 to 46.2 when the correlations are included.

After the population PWL value is determined by numerical integration, the simulation program then generates 4 sets of Marshall test results for the first project day. These 4 sets of test values are generated from a trivariate normal distribution with the parameters, i.e., means, standard deviations and correlation coefficients, input in Box 1. The payment factor for the first day (or lot) is then calculated by each of the methods. This process is then repeated for a total of 100 paving days for each project simulated.

Numerical Integration

The first method considered for establishing the payment factor for each lot was triple numerical integration (Box 4). This method considers the correlations between each of the variables along with the means and standard deviations. The PWL is computed in Box 4 and corresponds to the percentage of the total volume of the trivariate normal distribution that falls within the acceptance limits. This is

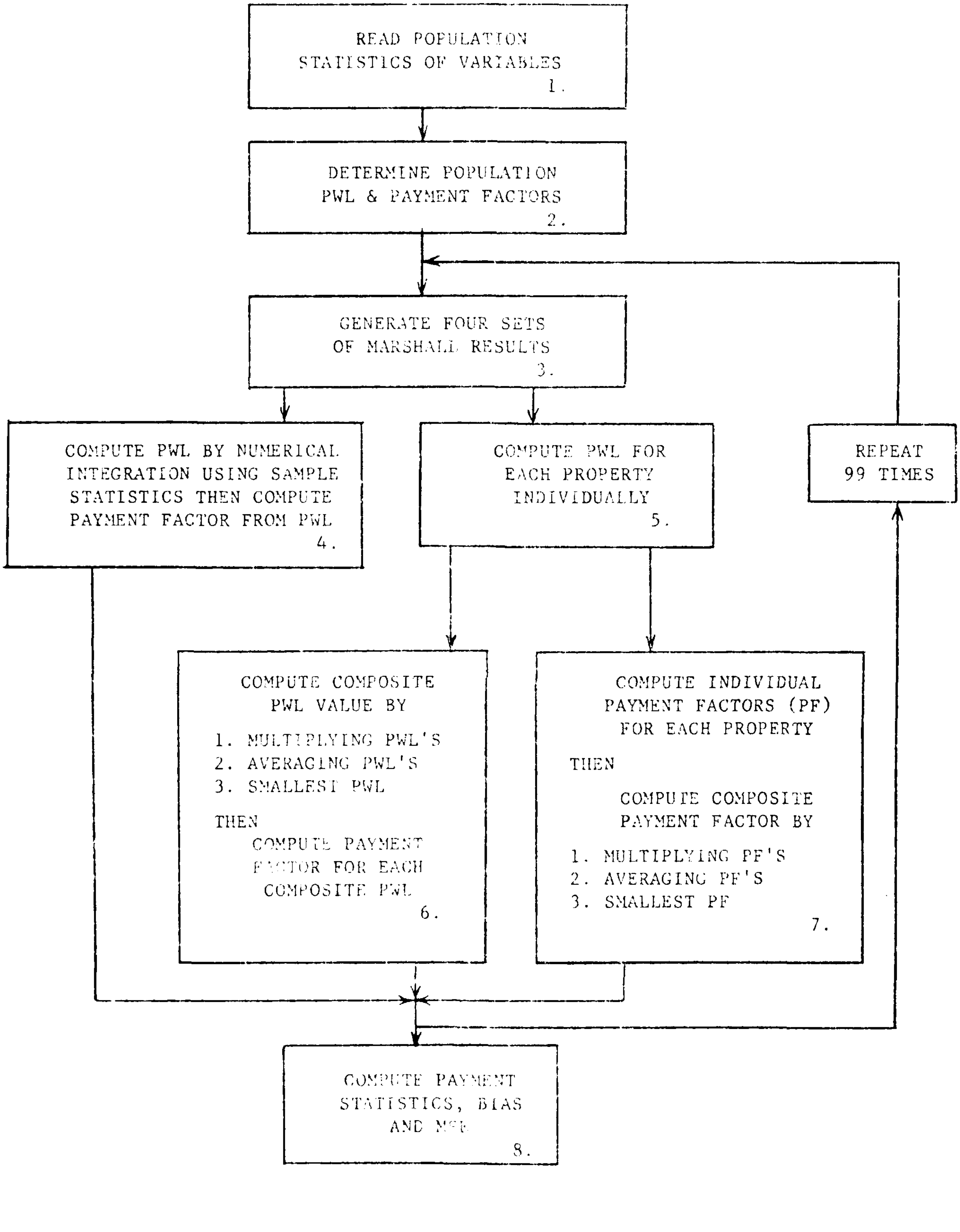


Figure 11. Flow Diagram [illegible]

# CHAPTER VI

# COMPUTER SIMULATION ANALYSIS

In Chapter V, several methods for determining the payment factor for the Marshall properties are presented. The current chapter presents the procedure and results from a computer simulation analysis to investigate the performance of the methods. Computer simulation is used to represent the Marshall test results for 100 lots of material for each project considered. Input to the simulation program is based on the data collected on the 15 runway paving projects discussed previously.

Four methods are considered for determining the payment factor for the Marshall properties. The methods examined include: 1) triple numerical integration, 2) multiplying the 3 individual property PWL values or payment factors, 3) averaging the 3 individual PWL values or payment factors and, 4) using the smallest of the 3 individual PWL values or payment factors.

## Simulation Procedures

Computer simulation was used to investigate the performance of the various methods (discussed in Chapter V) for determining the payment factor for multiple acceptance properties. Computer simulation was used to develop Marshall test results for 100 paving lots using the results from the 15 paving projects on which data were collected. The means, standard deviations and correlation coefficients from each of the 15 projects were used as the 'population' statistics in the various simulation analyses.

The results of 4 Marshall tests were generated for each paving day in the simulation analysis. The test values for the 3 correlated Marshall properties, i.e., stability, flow and air voids, were generated simultaneously by an algorithm, CSDA, based on Cholesky's Sequential Matrix Decomposition (see Chapter III). The simulated Marshall results were then used to determine the payment factor for the lot using each of the methods described previously. The procedures for determining these payment factors in the simulaton analyses are presented in the next section.

The simulation procedure is presented in the flow diagram of Figure 11. A detailed user's guide for the program is presented in Appendix C and a copmplete listing of the program appears in Appendix D. The 'population' statistics referred to in Box 1 of Figure 11 are those from the 15 runway paving projects from 1978 and 1981. The statistics input for each project simulation are the mean and standard deviation values for each of the 3 Marshall properties and the 3 correlaton coefficients. The values used in the simulations are presented in Tables 9-11 and 13-15.

b) Average pay factor - Mississippi Specifications - 401.22 B.

"The final percentage for each lot, any of which characteristics for asphalt content, gradation, density, and stability were not within reasonably close conformity, shall be determined as in the following example.

"Assume price adjustment for asphalt content is 90 percent, for the No. 200 sieve is 70 percent, for density is 100 percent and for stability is 100 percent.

"Thus the final pay factor for that lot would be:

$$\frac{90\% + 70\% + 100\% + 100\%}{4} = 90\% \text{ "}$$

c) Product of all pay factors - Nebraska Specifications - 507.13.

". . . that lots of asphalt concrete, accepted by the Engineer, shall be paid for at the contract unit price per ton for the item, . . . multiplied by product of the lot pay factors for asphalt content, retention on the applicable control sieves, and density of the compacted asphaltic concrete."

The numerical integration approaches presented previously include the correlation among the properties used when calculating the PWL values used to establish payment. The individual properties approaches do not directly consider the correlation which may exist among the 3 properties being considered. The payment factors product method essentially assumes that there is zero correlation among the properties. The payment factors averaging method assumes no functional correlation among the 3 properties, or that any correlation is somehow accounted for in the averaging process. The minimum payment factor approach implicitly assumes perfect correlation among the 3 variables.

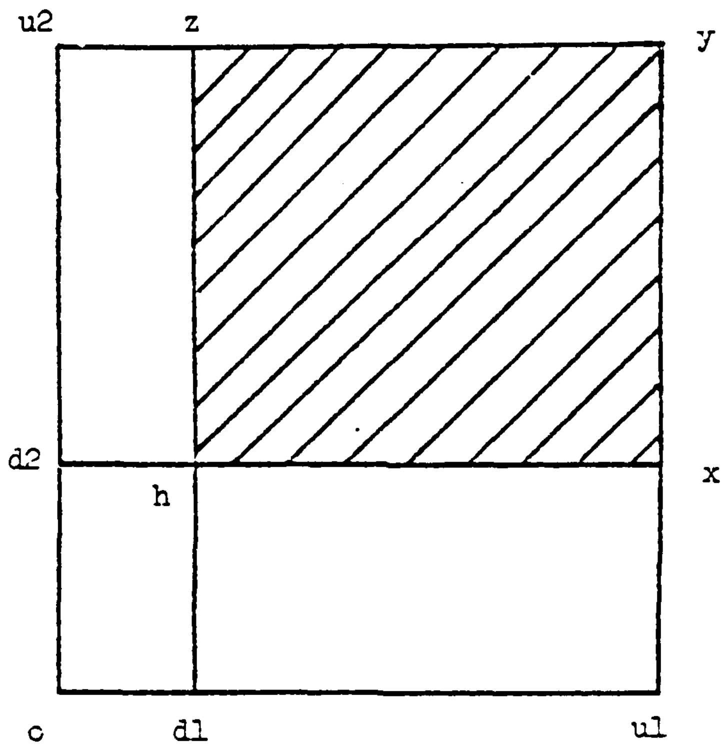


Figure 10. Illustration of the Bivariate PWL Expressed in Terms of PWL's with Variates Beginning at Minus Infinity.

standardized variates because the volume beyond these limits is negligible. Bivariate normal tables were developed that give the volume under the bivariate normal surface from -3.0 for both variates to a particular value of each standardized variate.

As shown in Figure 10, the area between the 4 specification limits which corresponds to the volume under the bivariate normal distribution (PWL) can be expressed in terms of 4 component volumes. The PWL (shaded region in Figure 10) can be expressed as the total area (o, u1, y, u2) minus the 2 side rectangles (o, d1, z, u2) and (o, d2, x, u1) plus the corner rectangle (o, d2, h, d1) which was subtracted twice. Hence, the PWL can be expessed as:

$$PWL = V(u1,u2) - V(u1,d2) - V(d1,u2) + V(d1,d2)$$

where u1 = upper limit for first variate,

u2 = upper limit for second variate,

d1 = lower limit for first variate,

d2 = lower limit for second variate.

The bivariate normal tables are given in Appendix B. Because the bivariate normal distribution variates are standardized, the tables are symmetrical, that is, V(u1,d2) is equal to V(d2,u1).

## Individual Properties Approaches

The bivariate approach still presents implementation problems at the present time. It is so different from current methods that resistance from field personnel is inevitable. In light of this, a number of approaches were also considered that are based on the same PWL estimation procedures currently employed by the FAA Eastern Region. These procedures consist of determining either a PWL or payment factor (PAY) for each of the properties individually, and then combining these in some fashion to arrive at a total PWL or PAY value for the lot. The approaches considered include: 1) multiplying the individual PWL or PAY values, 2) averaging the individual PWL or PAY values and, 3) using the smallest individual PWL or PAY value.

These approaches are similar to those currently employed by some state highway agencies that apply price adjustments for more than one characteristic. Examples of these methods, from Moore et. al. (10), are quoted below:

a) Lowest Pay factor - Georgia Specifications - 400.06 A.

"When two or more pay factors for a specific acceptance lot are less than 1.0, the adjusted payment will be determined by multiplying the contract unit price by the lowest pay factor."

Table 19. Mean Project Stability PWL Values for the 1978 Paving Projects

| Project | Marshall Stability | PWL |
|---|---|---|
| Adirondack-A | 2240.1 | 93.6 |
| Adirondack-B | 2341.6 | 98.3 |
| Charlottesville-ANJ | 2702.6 | 100.0 |
| Charlottesville-SLW | 3614.7 | 100.0 |
| Chautauqua | 2450.4 | 100.0 |
| Chemung-Chemung | 2427.3 | 100.0 |
| Chemung-Fisherville | 2475.9 | 99.5 |
| DuBois | 2056.1 | 93.1 |
| Dutchess | 2853.6 | 100.0 |
| Linden | 2117.9 | 99.4 |
| Westchester-Colprovia | 2816.1 | 100.0 |
| Westchester-Peckham | 2686.2 | 97.9 |

Table 20. Mean Project Stability PWL Values for the 1981 Paving Projects

| Project | Marshall Stability | PWL |
|---|---|---|
| Atlantic City | 2487.1 | 99.1 |
| Baltimore-Washington | 2744.0 | 100.0 |
| Rochester | 3207.3 | 100.0 |

160,000 tables would be needed to cover all the possible values of the variates and their correlations. The number of tables can be reduced significantly if the correlation values are limited to a few combinations, possibly around the trend values. However, such simplification limits the generality of this approach. Therefore, trivariate normal tables are not a practical alternative.

Bivariate Approach

The first step considered in an effort to simplify the procedures of the trivariate approach takes advantage of the fact that, for nearly all data collected on all projects, Marshall stability rarely failed to meet the acceptance requirements. Taking this into consideration, the problem can be reduced to a bivariae normal distribution by considering stability on an accept-or-reject basis, and using only flow and air voids for payment determination. This reduces to 19 the number of tables necessary to reasonably estimate PWL, and makes manual computations feasible.

By reducing the number of acceptance variables from 3 to 2, the number of statistics is reduced from 9 (3 means, 3 standard deviations and 3 correlations) to 5 (2 means, 2 standard deviations and 1 correlation).

The field data from both the 1978 and 1981 projects show that compliance with stability is more attainable than compliance with flow and air voids. All projects for both years have a mean project stability PWL well above 90 (Tables 19 and 20). Therefore, stability for most projects shold be accepted at full payment. Any major deficiency in the material's trivariate PWL is probably due to deficiencies in flow or air voids. Because flow and air voids are correlated, the bivariate normal distribution should be used to calculate PWL. Table 18 can then be used to obtain the adjusted payment factor.

The bivariate approach is much simpler and more practical to implement than the trivariate method, and does not have the same limitations.

As shown below, the determinant of the bivariate correlation matrix, $|C|$, is always positive definite since $-1 < P < 1$.

$$|C| = \begin{bmatrix} 1 & P \\ P & 1 \end{bmatrix} = 1 - P*P$$

Also, because acceptance will be based on only 2 variables, the number of statistics needed to generate tables is reduced to 3 (2 standardized variates and 1 correlation). Consequently, the number of tables for 31 standardized variate values (from -3.0 to +3.0 with 0.1 increments) and 19 correlation values (from -0.9 to +0.9 with 0.1 increments) is 19. These can be contained in a small handbook. The 2 numbers, -3.0 and +3.0, are used as lower and upper limits for the

method for claculating |C| is illustrated in the steps below.

$$[C] = \begin{bmatrix} 1 & C12 & C13 \\ C21 & 1 & C23 \\ C31 & C32 & 1 \end{bmatrix}$$

$$|C| = 1\ (1 - C23 * C23) - C12\ (C12 - C13 * C23) + C13\ (C12 * C23 - C13)$$

$$|C| = 1 + 2\ (C12 * C13 * C23) - (C12 * C12) - (C13 * C13) - (C23 * C23)$$

where C12 = correlation between stability and flow,
C13 = correlation between stability and air voids,
C23 = correlation between flow and air voids.

When all 3 correlations are large in magnitude and negative, |C| will be negative. For example, if C12 = C13 = C23 = -0.6, then the determinant, |C|, will be equal to -0.296. It is not likely that this situaton will often occur in practice.

2. Performing numerical integration requires access to a large digital computer. Nearly 71,000 statements are executed to calculate a single trivariate PWL. This problem can be overcome by installing a dial-up computer terminal at the project site to provide access to a central computer that can make the necessary computations. Alternatively, it may be possible to adapt the integration routine to operate on a micro-computer that can be located at the project office.

3. The computations needed to obtain sample statistics (3 means, 3 standard deviations and 3 correlation coefficients) from 4 Marshall tests is a fairly complicated procedure. However, computations can be performed easily by a programmable hand-held calculator or a micro-computer.

As indicated above, to implement triple integration the project sites should have, or have access to, computer facilities. Since this situation does not always exist, at least at the present time, alternatives must be considered. One such alternative is the use of trivariate normal tables. There are 8 variables (1 upper and 1 lower standardized variate for air voids and flow, 1 lower standardized variate for stability, and 3 correlation coefficients) that must be included in the tables. Assuming that each variable's range of values can be broken down into 20 intervals, 64 million (number of intervals, 20, raised to the 6th power, i.e., the number of variables less 2) tables will be needed to include all the possible values of these 8 variables. The number of variables can be reduced to 6 (3 standardized variates and 3 correlations) if a procedure which expresses the PWL as a function of 8 PWLs having variates without lower limits is used. Still,

Table 18. Current FAA Eastern Region Mat Density Payment Schedule

| Calculated PWL Value | Payment Factor (percentage) |
|---|---|
| 90 - 100 | 100 |
| 80 - 90 | 0.5 PWL + 55 |
| 65 - 80 | 2.0 PWL - 65 |
| Below 65 | 50 (or remove and replace) |

## CHAPTER V

## ACCEPTANCE AND PAYMENT PROCEDURES

With the preliminary analyses complete, the major area of the research effort was a computer simulation analysis to investigate the potential performance of multiple price adjustment approaches. Due to the complexity of the problem presented by the case of 3 acceptance criteria, it is necessary to use computer simulation to develop the operating characteristics for the proposed acceptance plans. The simulation effort consisted of sampling from a trivariate normal distribution and then calculating the payment level using several different payment determination procedures. Each of the methods considered used the current FAA density payment schedule shown in Table 18 to convert the estimated PWL values to payment factors.

A number of different approaches were considered for determining the acceptable payment for a lot of material based upon the 3 Marshall properties. These approaches can be divided into 2 major categories. The first category relates to approaches which consider the multivariate nature of the problem. The second category relates to methods which consider the 3 properties individually, and then incorporate the 3 values into a single (composite) payment level.

### Multivariate Approaches

#### Trivariate Approach

The most theoretically acceptable approach to use as a means of evaluating Marshall results for acceptance is based on synthesizing the 3 values, stability, flow and air voids, into a single number for acceptance purposes. This number is the percentage of the total volume of the trivariate normal distribution that falls within the acceptance limits (trivariate PWL). This is a logical extension of the single variable acceptance approach based on PWL currently employed by the FAA Eastern Region.

This approach uses 9 sample statistics calculated from the test results for each lot to estimate the PWL value for the lot. These values include the sample means and variances for stability, flow and air voids, and the 3 correlation coefficients.

There are several limitations and disadvantages to the use of triple numerical integration for acceptance calcu' ons. These limitations include:

1. The correlation matrix must be positive definite. In a few situations, the correlation matrix might not be positive definite, i.e., the determinant of the correlation matrix, $|C|$, might be negative. The

Table 17. Laboratory Correlation Coefficients Among the Marshall Properties (3)

| Gradation | Asphalt Content | Stability and Flow | Stability and Voids | Flow and Voids |
|---|---|---|---|---|
| A | 5.0 | 0.449 | -0.286 | 0.368 |
| | 5.5 | 0.522 | -0.329 | -0.010 |
| | 6.0 | 0.860 | -0.415 | -0.495 |
| | 6.5 | 0.644 | 0.378 | -0.137 |
| | 7.0 | 0.770 | 0.133 | -0.256 |
| | 7.5 | -0.263 | -0.619 | -0.023 |
| B | 5.0 | 0.645 | -0.532 | -0.220 |
| | 5.5 | 0.422 | 0.444 | 0.084 |
| | 6.0 | 0.116 | -0.200 | -0.682 |
| | 6.5 | 0.674 | -0.010 | -0.184 |
| | 7.0 | 0.692 | -0.401 | -0.148 |
| | 7.5 | 0.350 | -0.532 | -0.252 |
| C | 5.0 | 0.298 | -0.416 | -0.529 |
| | 5.5 | -0.223 | -0.396 | -0.308 |
| | 6.0 | 0.278 | -0.473 | -0.753 |
| | 6.5 | -0.005 | -0.113 | -0.767 |
| | 7.0 | -0.547 | -0.407 | 0.017 |
| | 7.5 | 0.540 | -0.631 | -0.345 |
| D | 5.0 | 0.006 | -0.156 | -0.147 |
| | 5.5 | 0.048 | -0.601 | 0.537 |
| | 6.0 | 0.282 | -0.643 | -0.493 |
| | 6.5 | 0.032 | -0.635 | -0.731 |
| | 7.0 | -0.260 | -0.515 | 0.306 |
| | 7.5 | 0.383 | -0.457 | -0.688 |

Table 15. Correlation Coefficients of the Marshall Results for the 1981 Field Data

| Project | Stability and Flow | Stability and Voids | Flow and Voids |
|---|---|---|---|
| Atlantic City | 0.069 | -0.334 | -0.301 |
| Baltimore-Washington | -0.601 | -0.296 | 0.075 |
| Rochester | 0.086 | -0.235 | -0.012 |

Table 16. PWLs of the Marshall Results for the 1981 Paving Projects

| Project | Number of tests | Marshall PWL |
|---|---|---|
| Atlantic City | 198 | 92.8 |
| Baltimore-Washington | 67 | 99.0 |
| Rochester | 53 | 99.3 |

accomplished by the same algorithm that was used in Box 2 to determine the population PWL value.

The limits of integration are the standardized acceptance limits. The sample statistics, XBAR and STD, used to calculate the standardized acceptance limits will vary for each project day. Therefore, the limits of integration will also vary. To reduce integration time, the largest upper and smallest lower integration limits for all variables are set at 3.5 and -3.5, respectively. Once the PWL value is calculated by the integration subroutine, the payment factor is determined from the schedule in Table 18.

Individual Properties

As indicated previously, several methods may be used to compute a composite payment factor based on the 3 individual Marshall properties. Each of these methods is based on determining either PWL values or payment factors for each of the properties individually, then combining the 3 individual values into a single composite payment factor. The methods considered in the computer simulation analyses for determining the composite payment factor can be divided into 2 categories. The first category consists of those methods that combine the individual PWL values into a single PWL value that is then used to determine the payment factor from Table 18. The second category consists of those methods that detemine a payment factor for each individual property and then combine the payment factors into a single composite payment factor.

Three methods for obtaining a composite value were used with each category described in the previous paragraph. These methods include: 1) multiplying the individual PWL values or payment factors, 2) averaging the individual PWL values or payment factors and 3) using the smallest of the individual PWL values or payment factors. These procedures are shown in the flow chart in Figure 11.

Once the 4 sets of Marshall test results are generated in Box 3, the sample means and standard deviations are used to determine the 3 individual property PWL values by the quality index approach currently used by the FAA Eastern Region (Box 5). In Box 6, the composite PWL value is obtained by each of the 3 methods presented above. The payment factor is then determined from Table 18 using the composite PWL value. In Box 7, individual payment factors are first determined from Table 18 for each of the 3 properties. The individual payment factors are then combined into composite payment factors by each of the 3 methods described above.

The procedures outlined in Boxes 3-7 are repeated a total of 100 times for each set of 'population' statistics, i.e., projects, considered. The results for the 100 project days are then summarized in Box 8. The results from Box 8 can be used to evaluate the performance of each of the payment determination methods investigated.

## Evaluation Criterion

Two important factors to be considered when evaluating an estimator are the bias and variability of the estimator. The variability of the estimator is represented by the variance. By way of definition, an estimator is unbiased if its expected value is the same as the parameter (in this cae, the payment factor) it is being used to estimate. In the simulation analyses, the mean square error (MSE) of a payment determination method is used as the norm and the minimum MSE as the criterion for choice between the methods. The MSE norm is chosen because it incorporates the 2 important measures of bias and variance. The MSE for a method can be described as follows:

if

$$\text{Bias} = E[\text{PHAT}(i)] - \text{PAY}$$

and

$$\text{Var}[\text{PHAT}(i)] = E[\text{PHAT}(i) - E(\text{PHAT})]^2$$

then

$$\text{MSE} = E[\text{PHAT}(i) - \text{PAY}]^2 = \text{Var}[(\text{PHAT}(i)] + (\text{Bias})^2$$

| | | |
|---|---|---|
| where: | PAY | = correct payment factor for the poputlation |
| | i | = payment determination method, i = 1,2,....7 |
| | PHAT(i) | = estimated payment factor using method i |
| | E[PHAT(i)] | = expected payment factor for method i |
| | E(PHAT) | = expected value of the payment factor |
| | Var | = variance of the term in the brackets. |

For computational purposes, the sampling equivalents of the expectations, with the averages being taken over the 100 replications, are used.

A total of 7 payment determination methods are evaluated with the MSE criterion in the simulation analyses. These 7 methods include:

1) triple numerical integration using the daily sample means, standard deviations and correlation coefficients,

2) multiplying the individual PWL values to obtain a composite PWL value,

3) averaging the individual PWL values to obtain a composite PWL value,

4) using the smallest individual PWL value as the composite PWL value,

5) multiplying the individual payment factors to obtain a composite payment factor,

6) averaging the individual payment factors to obtain a composite payment factor and

7) using the smallest individual payment factor as the composite payment factor.

## SIMULATION RESULTS

The results for each project simulated are the payment statistics (the mean and variance of the 100 payment factors), the Bias and the MSE. The results for the simulation exercise are given in Tables 22 and 23. Table 22 presents the results of the 1978 simulated projects. Table 23 presents the results for the 1981 projects along with the results using the pooled means and standard deviations from the 3 projects and the trend correlations. For each project in the tables, the mean and variance of the 100 payment factors and the Bias and MSE values are shown for each of the 7 payment determination methods.

### Analysis of the Results

The results in Tables 22 and 23 indicate that the performance of the payment determination methods is influenced by the quality of the material being evaluated. For projects with low quality, and therefore low population payment factors (e.g., Adirondack-A, Charlottesville-ANJ and -SLW, and Westchester-Peck with true payment factors of 50), the triple integration approach provided less Bias and smaller MSE values. For projects with high quality and high population payment factors (e.g., the 8 projects with true payment factors of 100), the averaging methods, methods 3 and 6, provide the superior results. As noted previously, the effect of correlaton among the variables is relatively small for populations with high PWL values. This may explain why the averaging methods work well on the projects which have high PWL values (as evidenced by the pay factors of 100).

The low expected payment factors, the resulting large negative biases and the high MSE values for the integration method (Method 1) can be associated with 2 factors. The first is the fact that the small number of samples per day (i.e., 4) does not provide a very good estimate for the population correlaton values. That is, the correlation coefficients calculated for sample sizes of 4 may vary greatly from the values of the populaion from which they were drawn. Since these coefficients are used in the triple integration routine, the PWL values resulting from the integration will also tend to be highly variable. This fact is reflected in the high variance and MSE values for Method 1 in Tables 22 and 23.

The second factor affecting the performance of the integration method relates to the payment schedule that is used. The payment schedule (Table 18) was developed for the single variable case of mat density. As currently implemented by the FAA Eastern Region, on an acceptance but not price determination basis, the Marshall properties are considered to be in substantial compliance with the specifications if the individual PWL values for each of the 3 properties are 90 or

Table 22. Results of Computer Simulation Analyses

| Project | Method* | E[PHAT]# | Var[PHAT]@ | Bias | MSE |
|---|---|---|---|---|---|
| Adirondack-A | 1 | 57.5 | 253.9 | 7.5 | 310.2 |
| | 2 | 85.9 | 381.5 | 35.9 | 1670.7 |
| (50.0)$ | 3 | 98.9 | 10.3 | 48.9 | 2403.8 |
| | 4 | 91.2 | 176.7 | 41.2 | 1875.9 |
| | 5 | 90.5 | 216.2 | 40.5 | 1854.8 |
| | 6 | 96.7 | 28.2 | 46.7 | 2208.8 |
| | 7 | 91.2 | 176.7 | 41.2 | 1875.9 |
| Adirondack-B | 1 | 66.5 | 423.6 | -29.1 | 1268.1 |
| | 2 | 93.6 | 154.3 | - 1.9 | 157.8 |
| (95.5) | 3 | 99.8 | 0.9 | 4.3 | 19.0 |
| | 4 | 95.4 | 104.9 | - 0.1 | 104.9 |
| | 5 | 95.2 | 114.1 | - 0.4 | 114.2 |
| | 6 | 98.4 | 13.3 | 2.8 | 21.4 |
| | 7 | 95.4 | 104.9 | - 0.1 | 104.9 |
| Charlottesville-ANJ | 1 | 51.5 | 51.2 | 1.5 | 53.5 |
| | 2 | 54.7 | 157.8 | 4.7 | 179.6 |
| (50.0) | 3 | 87.5 | 193.7 | 37.5 | 1599.6 |
| | 4 | 56.3 | 205.3 | 6.3 | 245.4 |
| | 5 | 51.1 | 323.7 | 1.1 | 324.9 |
| | 6 | 82.0 | 64.6 | 32.0 | 1089.7 |
| | 7 | 56.3 | 205.3 | 6.3 | 245.4 |
| Charlottesville-SLW | 1 | 52.2 | 57.2 | 2.2 | 62.1 |
| | 2 | 58.6 | 271.5 | 8.6 | 345.5 |
| (50.0) | 3 | 92.0 | 113.3 | 42.0 | 1876.4 |
| | 4 | 63.2 | 342.1 | 13.2 | 516.7 |
| | 5 | 59.4 | 435.2 | 9.4 | 524.1 |
| | 6 | 85.5 | 69.8 | 35.5 | 1326.7 |
| | 7 | 63.2 | 342.1 | 13.2 | 516.7 |

# - expected payment factor     @ - variance of payment factor

$ - correct payment factor for the population

* - payment determination method:

1. triple numerical integration
2. multiplying the individual PWL values
3. averaging the individual PWL values
4. using the smallest individual PWL value
5. multiplying the individual payment factors
6. averaging the individual payment factors
7. using the smallest individual payment factor

Table 22. Results of Computer Simulation Analyses (continued)

| Project | Method* | E[PHAT]# | Var[PHAT]@ | Bias | MSE |
|---|---|---|---|---|---|
| Chautauqua | 1 | 85.3 | 366.0 | -14.7 | 582.7 |
| | 2 | 99.0 | 28.5 | - 1.0 | 29.5 |
| (100)$ | 3 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 4 | 99.0 | 28.5 | - 1.0 | 29.5 |
| | 5 | 99.0 | 28.5 | - 1.0 | 29.5 |
| | 6 | 99.7 | 3.2 | - 0.3 | 3.3 |
| | 7 | 99.0 | 28.5 | - 1.0 | 29.5 |
| Chemung-Chem | 1 | 79.9 | 428.3 | -20.1 | 833.0 |
| | 2 | 98.9 | 30.1 | - 1.1 | 31.3 |
| (100) | 3 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 4 | 98.9 | 30.1 | - 1.1 | 31.3 |
| | 5 | 98.9 | 30.1 | - 1.1 | 31.3 |
| | 6 | 99.6 | 3.4 | - 0.4 | 3.5 |
| | 7 | 98.9 | 30.1 | - 1.1 | 31.3 |
| Chemung-Fish | 1 | 69.9 | 437.0 | -27.4 | 1189.4 |
| | 2 | 91.3 | 207.4 | - 6.0 | 243.8 |
| (97.3) | 3 | 99.8 | 0.5 | 2.5 | 6.7 |
| | 4 | 91.7 | 203.4 | - 5.7 | 235.4 |
| | 5 | 91.6 | 204.1 | - 5.7 | 236.3 |
| | 6 | 97.2 | 22.8 | - 0.1 | 22.8 |
| | 7 | 91.7 | 203.4 | - 5.7 | 235.4 |
| Dubois | 1 | 60.2 | 311.1 | -25.6 | 966.7 |
| | 2 | 91.1 | 241.8 | 5.3 | 270.4 |
| (85.8) | 3 | 99.4 | 4.7 | 13.7 | 191.4 |
| | 4 | 95.2 | 107.0 | 9.4 | 195.3 |
| | 5 | 94.7 | 128.3 | 9.0 | 208.5 |
| | 6 | 98.2 | 16.0 | 12.4 | 169.8 |
| | 7 | 95.2 | 107.0 | 9.4 | 195.3 |

# - expected payment factor @ - variance of payment factor

$ - correct payment factor for the population

* - payment determination method:

1. triple numerical integration
2. multiplying the individual PWL values
3. averaging the individual PWL values
4. using the smallest individual PWL value
5. multiplying the individual payment factors
6. averaging the individual payment factors
7. using the smallest individual payment factor

Table 22. Results of Computer Simulation Analyses (continued)

| Project | Method* | E[PHAT]# | Var[PHAT]@ | Bias | MSE |
|---|---|---|---|---|---|
| Dutchess | 1 | 75.6 | 387.3 | -23.8 | 953.1 |
| | 2 | 95.8 | 87.9 | - 3.5 | 100.2 |
| (99.3)$ | 3 | 99.9 | 0.3 | 0.6 | 0.6 |
| | 4 | 96.9 | 39.5 | - 2.5 | 45.6 |
| | 5 | 96.7 | 46.6 | - 2.6 | 53.5 |
| | 6 | 98.9 | 5.6 | - 0.5 | 5.8 |
| | 7 | 96.9 | 39.5 | - 2.5 | 45.6 |
| Linden | 1 | 77.8 | 411.0 | -22.2 | 903.5 |
| | 2 | 98.1 | 44.3 | - 1.9 | 48.1 |
| (100) | 3 | 100.0 | 0.1 | 0.0 | 0.1 |
| | 4 | 98.3 | 27.7 | - 1.7 | 30.5 |
| | 5 | 98.3 | 28.8 | - 1.7 | 31.7 |
| | 6 | 99.4 | 3.3 | - 0.6 | 3.6 |
| | 7 | 98.3 | 27.7 | - 1.7 | 30.5 |
| Westchester-Colp | 1 | 75.1 | 428.7 | -24.9 | 1046.5 |
| | 2 | 97.6 | 70.0 | - 2.4 | 75.6 |
| (100) | 3 | 100.0 | 0.1 | 0.0 | 0.1 |
| | 4 | 97.9 | 62.6 | - 2.1 | 67.1 |
| | 5 | 97.9 | 62.6 | - 2.1 | 67.1 |
| | 6 | 99.3 | 7.0 | - 0.7 | 7.5 |
| | 7 | 97.9 | 62.6 | - 2.1 | 67.1 |
| Westchester-Peck | 1 | 54.7 | 158.5 | 4.7 | 180.9 |
| | 2 | 73.5 | 500.9 | 23.5 | 1054.1 |
| (50.0) | 3 | 95.6 | 77.4 | 45.6 | 2153.0 |
| | 4 | 79.2 | 398.5 | 29.2 | 1252.0 |
| | 5 | 75.6 | 590.8 | 25.6 | 1246.6 |
| | 6 | 90.9 | 95.2 | 40.9 | 1771.0 |
| | 7 | 79.2 | 398.5 | 29.2 | 1252.0 |

# - expected payment factor  @ - variance of payment factor

$ - correct payment factor for the population

* - payment determination method:

1. triple numerical integration
2. multiplying the individual PWL values
3. averaging the individual PWL values
4. using the smallest individual PWL value
5. multiplying the individual payment factors
6. averaging the individual payment factors
7. using the smallest individual payment factor

Table 23. Results of Computer Simulation Analyses - 1981 Projects

| Project | Method* | E[PHAT]# | Var[PHAT]@ | Bias | MSE |
|---|---|---|---|---|---|
| Atlantic City | 1 | 75.8 | 397.1 | -24.2 | 981.3 |
| | 2 | 96.9 | 60.2 | - 3.1 | 69.7 |
| (100)$ | 3 | 99.9 | 0.1 | - 0.1 | 0.1 |
| | 4 | 97.8 | 25.3 | - 2.2 | 30.0 |
| | 5 | 97.8 | 27.1 | - 2.2 | 32.0 |
| | 6 | 99.3 | 3.1 | - 0.7 | 3.7 |
| | 7 | 97.8 | 25.3 | - 2.2 | 30.0 |
| Baltimore-Washington | 1 | 90.2 | 257.9 | - 9.8 | 354.7 |
| | 2 | 100.0 | 0.0 | 0.0 | 0.0 |
| (100) | 3 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 4 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 5 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 6 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 7 | 100.0 | 0.0 | 0.0 | 0.0 |
| Rochester | 1 | 87.4 | 330.7 | -12.6 | 488.5 |
| | 2 | 100.0 | 0.1 | 0.0 | 0.1 |
| (100) | 3 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 4 | 100.0 | 0.1 | 0.0 | 0.1 |
| | 5 | 100.0 | 0.1 | 0.0 | 0.1 |
| | 6 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 7 | 100.0 | 0.1 | 0.0 | 0.1 |
| Pooled | 1 | 79.9 | 449.7 | -20.1 | 852.8 |
| | 2 | 99.9 | 0.3 | - 0.1 | 0.3 |
| (100) | 3 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 4 | 99.9 | 0.3 | - 0.1 | 0.3 |
| | 5 | 99.9 | 0.3 | - 0.1 | 0.3 |
| | 6 | 100.0 | 0.0 | 0.0 | 0.0 |
| | 7 | 99.9 | 0.3 | - 0.1 | 0.3 |

# - expected payment factor    @ - variance of payment factor

$ - correct payment factor for the population

* - payment determination method:

1. triple numerical integration
2. multiplying the individual PWL values
3. averaging the individual PWL values
4. using the smallest individual PWL value
5. multiplying the individual payment factors
6. averaging the individual payment factors
7. using the smallest individual payment factor

greater. If the 3 properties are considered to be non-correlated, then this is equivalent to requiring a PWL value of 72.9 (.9 x .9 x .9 x 100) for all 3 properties considered simultaneously. As shown in Table 21, this number could be even lower if the correlations between the properties are considered. If the payment schedule in Table 18 were modified to require a lower PWL value for full payment, then the large negative biases for Method 1 would be reduced in Tables 22 and 23.

Such a modification in the payment schedule for the multiple Marshall properties would probably reduce the large negative biases, but would not improve the high variability introduced by the estimate for the correlation coefficients nor would it reduce the large implementation problems associated with such a drastic change from current practices. It would therefore seem that the individual properties approaches are preferred over the integration method even though the integration method appears to be the most theoretically sound of the methods considered.

To investigate the effect of the number of samples on the integration method, a sensitivity analysis was conducted in which the number of samples per day was allowed to vary from 4 to 20. Table 24 presents the results of this analysis. The pooled values for means and standard deviatons for the 1981 projects were used with the trend correlations for the sensitivity analysis. As seen in Table 24, the Bias is reduced from -20.1 to -0.2 as the number of samples is increased from 4 to 20. At the same time, the MSE is reduced from 852.8 to 0.5. However, it is not reasonable to expect to obtain more than 4 Marshall tests per day due to the time and expense involved. This establishes a practical limit on the number of samples that can be obtained and tested in a paving day.

## Selection of the Payment Determination Method

With the integration approach eliminated, the selection of the payment determination method is reduced to determining which of the 3 composite approaches, i.e., multiplying, averaging or using the smallest individual value, provides the smallest MSE values. It is also necessary to determine whether the method chosen should be applied to the individual PWL values or to the individual payment factors. As noted previously, the averaging method provides superior results for population payment values near 100. Since 9 of the 15 projects considered in the study had populaton payment factor values greater than 97, the averaging method was selected as the method for combining the individual property values.

It was next necessary to select between averaging the individual PWL values, and averaging the individual payment factors to determine the composite payment factor. Table 25 presents a summary of the MSE results for averaging the PWL values (Method 3) and for averaging the payment factors (Method 6) for each of the projects simulated. As shown in the table, there is little difference in the MSE values for the 2 methods when the MSE values are small. However, the PWL averaging method (Method 3) does have consistently lower values in this range.

Table 24. Results of Sensitivity Analysis of the Effect of the Number of Samples on the Bias and MSE Results for the Integration Method. (Pooled Means and Standard Deviations and Trend Correlations)

| Number of Samples | Bias | MSE |
|---|---|---|
| 4 | -20.1 | 852.8 |
| 6 | - 6.5 | 210.6 |
| 8 | - 1.8 | 42.3 |
| 10 | - 0.5 | 1.8 |
| 12 | - 0.7 | 5.5 |
| 14 | - 0.5 | 2.0 |
| 16 | - 0.3 | 1.1 |
| 18 | - 0.2 | 0.5 |
| 20 | - 0.2 | 0.5 |

Table 25. MSE Values for Methods 3 and 6 for the Projects Simulated

| Project | Population Payment Factor | MSE (Method 3)* | MSE (Method 6)* |
|---|---|---|---|
| Adirondack-A | 50.0 | 2403.8 | 2208.8 |
| Adirondack-B | 95.5 | 19.0 | 21.4 |
| Charlottesville-ANJ | 73.7 | 1599.6 | 1089.7 |
| Charlottesville-SLW | 50.0 | 1876.4 | 1326.7 |
| Chautauqua | 100.0 | 0.0 | 3.3 |
| Chemung-Chem | 100.0 | 0.0 | 3.5 |
| Chemung-Fish | 97.3 | 6.7 | 22.8 |
| Dubois | 85.3 | 191.4 | 169.8 |
| Dutchess | 99.4 | 0.6 | 5.8 |
| Linden | 100.0 | 0.1 | 3.6 |
| Westchester-Colp | 100.0 | 0.1 | 7.5 |
| Westchester-Peck | 50.0 | 2153.0 | 1771.0 |
| Atlantic City | 100.0 | 0.1 | 3.7 |
| Baltimore-Washington | 100.0 | 0.0 | 0.0 |
| Rochester | 100.0 | 0.0 | 0.0 |
| Pooled | 100.0 | 0.0 | 0.0 |

* Method 3 - averaging the individual PWL values
Method 6 - averaging the individual payment factors

When the MSE values are very large (for low population payment factors) the payment factor averaging method (Method 6) has consistently lower MSE values.

It is difficult to select between the 2 averaging methods since niether is consistently superior for the entire range of population payment factors found on the projects studied. The individual payment factor averaging method is closer to the method currently being employed for calculating mat density payments. As a result, it may be more readily implemented and accepted by the parties involved on actual projects. For this reason it is recommended as the payment determination method for the Marshall properties.

## Marshall-Density Payment Factor Determination

One factor to be considered in the application of the composite Marshall poroperties payment factor is the relationship between this payment factor and the one calculated for density. If a strong correlation exists between the Marshall properties obtained in the laboratory and the density obtained in the field, then it might be necessary to consider this correlation when determining the overall payment factor for the lot of material. It is not possible to determine directly whether such correlations exist because there is no one-to-one correspondence between the Marshall test results and the field density test results.

The 4 sets of Marshall laboratory densities are averaged to determine the single value against which the individual field density results are measured. There is, therefore, no way to measure correlation values between the individual field density results and the individual Marshall results. Any potential effects of the Marshall properties on the compactability of the asphalt mixture should affect both the laboratory density and the field density. Since the field density is calculated as a percentage of the laboratory density, any caompactability effects associated with the Marshall properties should be accounted for in the calculation of the field density values. It would therefore not seem to be necessary to consider any correlation between density and the Marshall properties when determining the overall payment factor for the lot.

The same 3 approaches that were considered for establishing a composite payment factor for the Marshall properties can be considered for combining the Marshall payment factor and the density payment factor. These approaches are: 1) multiplying the 2 payment factors, 2) averaging the 2 payment factors and 3) using the smaller of the 2 payment factors. It is not clear which of these approaches is appropriate because it is not possible to clearly establish the correlation relationship between the Marshall and density results. Using the averaging approach would be consistent with averaging the 3 Marshall properties to determine the Marshall properties payment factor. It might also be thought of as being consistent with averaging the 4 Marshall laboratory density values against which to measure the individual field density tests.

# CHAPTER VII

## SUMMARY AND CONCLUSIONS

This report presents the procedure and results of a computer simulation analysis to investigate the performance of a number of methods for determining the payment factor for a lot of materials when 3 acceptance characteristics, i.e., the Marshall stability, flow and air voids, are employed. Although the Marshall properties were specifically addressed in this study, the procedures developed can be applied to the case of any 3 correlated properties. A total of 7 different methods were considered for calculating the payment factor. These methods include:

1) triple numerical integration to determine the overall PWL
2) multiplying the individual property PWL values
3) averaging the individual property PWL values
4) using the smallest individual property PWL value
5) multiplying the individual property payment factors
6) averaging the individual property payment factors
7) using the smallest individual property payment factor.

Marshall test results from 15 runway paving projects were analyzed to determine the means, variances and correlation coefficients (among the Marshall properties) that are obtained on actual construction projects. These results were used as input to numerical integration and computer simulaion routines to evaluate the 7 payment determination methods considered.

A numerical integration algorithm for the trivariate normal distribution was developed to calculate the total volume of the material that fell within the acceptance limits for each lot of material. To verify the integrity of the algorithm, integration results for the zero correlation case were compared with results using tables of the standard normal distribution. The integration algorithm agreed closely with the results derived from the tables.

Computer simulation was used to investigate the performance of the various methods for determining the payment factor for multiple acceptance properties. Computer simulation was used to develop Marshall test reslts for 100 paving lots using the results from the 15 paving projects for which data were collected. The mean square payment error (MSE) for each of the methods was used as the norm and the minimum MSE as the criterion for choice among the methods.

## Research Findings

The following findings can be noted from the research:

1) there are correlations between the Marshall properties that are statistically significantly different from zero,

2) numerical integration can be used to determine the overall percentage of a population of material that falls within the Marshall properties acceptance limits,

3) numerical integration with a digital computer is necessary to determine the trivariate PWL value for the Marshall properties since millions of tables would be required if numerical integration were not used,

4) if stability is considered on an accept-or-reject basis, then 19 tables are necessary to estimate the bivariate PWL for flow and air voids,

5) in the simulation analyses, the numerical integration method provided very large MSE values as compared with the individual properties methods because the small sample size, i.e., n=4, does not provide a good estimate of the population correlation coefficients that must be used in the integration algorithm,

6) the averaging method provides the smallest MSE values for populations with high PWL values (this was the case for the majority of the projects for which data were available),

7) there is little difference between the MSE values for the 2 averaging methods (i.e., averaging the PWL values or averaging the payment factors),

8) the method which averages the individual property payment factors is the one most similar to the method currently employed by the FAA Eastern Region for mat density payment factor determination.

## Recommendation

The payment determination procedure that is recommended for the trivariate case of the Marshall properties is the individual property payment factor averaging method. It is difficult to select between the 2 averaging methods since neither is consistently superior for the entire range of population payment factors found on the projects studied. The individual payment factor averaging method is closer to the method currently being employed for calculating mat density payments. As a result, it may be more readily implemented and accepted by the parties involved on actual projects. For this reason it is recommended as the payment determination method for the Marshall properties.

The following acceptance procedure for determining the payment factor for the Marshall properties is recommended:

1. Using the random sampling procedures in the FAA Eastern Region Laboratory Procedures Manual, select 4 samples from each lot of material for Marshall properties determination.

2. For each Marshall property, i.e., stability, flow and air voids, determine the PWL value using the Quality Index approach outlined in the Eastern Region P-401 specification.

3. Using the calculated PWL values and Table 18, determine the payment factor individually for each of the 3 Marshall properties.

4. The composite payment factor associated with the Marshall properties is then calculated as the average of the 3 individual payment factors.

5. The payment factor for density is calculated using Table 18 and the estimated PWL value determined by the Quality Index approach outlined in the Eastern Region P-401 specification.

6. The overall payment factor for the lot of material is calculated as the average of the Marshall properties payment factor and the density payment factor.

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BIVARIATE NORMAL TABLES

P=-0.1

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 13 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 14 | 15 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 9 | 12 | 14 | 17 | 20 | 22 | 25 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | [illegible] | 6 | 8 | 11 | 15 | 18 | 22 | 27 | 31 | 35 | 39 | 42 | 45 | 47 | 49 | 51 | 52 | 52 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 9 | 13 | 17 | 22 | 28 | 34 | 40 | 46 | 53 | 58 | 63 | 67 | 71 | 73 | 75 | 77 | 78 | 78 | 79 | 79 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 4 | 6 | 10 | 14 | 19 | 25 | 32 | 40 | 49 | 58 | 67 | 76 | 84 | 91 | 97 | 101 | 105 | 108 | 110 | 111 | 112 | 113 | 113 | 113 |
| -1.0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 14 | 19 | 27 | 35 | 46 | 57 | 69 | 82 | 94 | 106 | 117 | 126 | 134 | 141 | 146 | 149 | 152 | 154 | 155 | 156 | 157 | 157 |
| -0.8 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 8 | 13 | 19 | 27 | 36 | 48 | 62 | 77 | 94 | 111 | 127 | 143 | 157 | 170 | 180 | 189 | 195 | 200 | 204 | 206 | 208 | 209 | 210 | 210 |
| -0.6 | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 11 | 17 | 25 | 35 | 48 | 64 | 82 | 102 | 123 | 145 | 166 | 187 | 205 | 221 | 235 | 246 | 254 | 260 | 265 | 268 | 270 | 271 | 272 | 272 |
| -0.4 | 0 | 0 | 1 | 2 | 3 | 6 | 9 | 15 | 22 | 32 | 46 | 62 | 82 | 105 | 130 | 157 | 184 | 211 | 237 | 259 | 279 | 296 | 310 | 320 | 328 | 333 | 337 | 339 | 341 | 342 | 342 |
| -0.2 | 0 | 0 | 1 | 2 | 4 | 7 | 12 | 18 | 28 | 40 | 57 | 77 | 102 | 130 | 161 | 194 | 227 | 260 | 291 | 319 | 343 | 363 | 379 | 392 | 401 | 407 | 412 | 415 | 416 | 418 | 418 |
| 0.0 | 0 | 0 | 1 | 3 | 5 | 9 | 14 | 22 | 34 | 49 | 69 | 94 | 123 | 157 | 194 | 233 | 273 | 312 | 348 | 381 | 410 | 433 | 452 | 466 | 477 | 485 | 490 | 493 | 495 | 497 | 497 |
| 0.2 | 0 | 1 | 2 | 3 | 6 | 10 | 17 | 27 | 40 | 58 | 82 | 111 | 145 | 184 | 227 | 273 | 319 | 364 | 406 | 444 | 476 | 503 | 525 | 541 | 554 | 562 | 568 | 572 | 574 | 576 | 576 |
| 0.4 | 0 | 1 | 2 | 4 | 7 | 12 | 20 | 31 | 46 | 67 | 94 | 127 | 166 | 211 | 260 | 312 | 364 | 415 | 462 | 504 | 541 | 571 | 595 | 614 | 627 | 637 | 643 | 647 | 650 | 651 | 652 |
| 0.6 | 0 | 1 | 2 | 4 | 8 | 14 | 22 | 35 | 53 | 76 | 106 | 143 | 187 | 237 | 291 | 348 | 406 | 462 | 514 | 561 | 601 | 634 | 660 | 680 | 695 | 705 | 712 | 717 | 720 | 721 | 722 |
| 0.8 | 0 | 1 | 2 | 5 | 9 | 15 | 25 | 39 | 58 | 84 | 117 | 157 | 205 | 259 | 319 | 381 | 444 | 504 | 561 | 611 | 654 | 690 | 718 | 740 | 756 | 767 | 774 | 779 | 782 | 784 | 785 |
| 1.0 | 0 | 1 | 3 | 5 | 10 | 17 | 27 | 42 | 63 | 91 | 126 | 170 | 221 | 279 | 343 | 410 | 476 | 541 | 601 | 654 | 700 | 738 | 768 | 791 | 807 | 819 | 827 | 832 | 835 | 837 | 638 |
| 1.2 | 0 | 1 | 3 | 6 | 10 | 18 | 29 | 45 | 67 | 97 | 134 | 180 | 235 | 296 | 363 | 433 | 503 | 571 | 634 | 690 | 738 | 777 | 809 | 832 | 849 | 861 | 870 | 875 | 878 | 880 | 681 |
| 1.4 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 47 | 71 | 101 | 141 | 189 | 246 | 310 | 379 | 452 | 525 | 595 | 660 | 718 | 768 | 809 | 841 | 865 | 883 | 895 | 904 | 909 | 912 | 914 | 915 |
| 1.6 | 0 | 1 | 3 | 6 | 11 | 20 | 32 | 49 | 73 | 105 | 146 | 195 | 254 | 320 | 392 | 466 | 541 | 614 | 680 | 740 | 791 | 832 | 865 | 890 | 908 | 921 | 929 | 935 | 938 | 940 | 941 |
| 1.8 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 51 | 75 | 108 | 149 | 200 | 260 | 328 | 401 | 477 | 554 | 627 | 695 | 756 | 807 | 849 | 883 | 908 | 926 | 939 | 948 | 953 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 52 | 77 | 110 | 152 | 204 | 265 | 333 | 407 | 485 | 562 | 637 | 705 | 767 | 819 | 861 | 895 | 921 | 939 | 952 | 961 | 966 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 154 | 206 | 268 | 337 | 412 | 490 | 568 | 643 | 712 | 774 | 827 | 870 | 904 | 929 | 948 | 961 | 970 | 975 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 208 | 270 | 339 | 415 | 493 | 572 | 647 | 717 | 779 | 832 | 875 | 909 | 935 | 953 | 966 | 975 | 981 | 984 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 416 | 495 | 574 | 650 | 720 | 782 | 835 | 878 | 912 | 938 | 957 | 970 | 979 | 984 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 651 | 721 | 784 | 837 | 880 | 914 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 13 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 785 | 838 | 881 | 915 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
$\rho$ 0.0

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 13 | 13 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 5 | 6 | 7 | 9 | 11 | 12 | 14 | 16 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 9 | 12 | 15 | 17 | 20 | 23 | 25 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 8 | 11 | 15 | 18 | 22 | 27 | 31 | 35 | 39 | 42 | 45 | 47 | 49 | 50 | 51 | 52 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 9 | 12 | 17 | 22 | 27 | 33 | 40 | 46 | 52 | 58 | 62 | 67 | 70 | 73 | 75 | 76 | 77 | 78 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 13 | 18 | 24 | 31 | 39 | 48 | 57 | 66 | 74 | 82 | 89 | 96 | 100 | 104 | 107 | 109 | 111 | 112 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 12 | 18 | 25 | 33 | 43 | 54 | 66 | 78 | 91 | 103 | 114 | 124 | 132 | 139 | 144 | 148 | 151 | 154 | 155 | 156 | 156 | 157 | 157 |
| -0.8 | 0 | 0 | 1 | 1 | 3 | 5 | 7 | 11 | 17 | 24 | 33 | 44 | 57 | 72 | 88 | 105 | 122 | 138 | 152 | 166 | 177 | 186 | 193 | 199 | 203 | 205 | 207 | 208 | 209 | 210 | 210 |
| -0.6 | 0 | 0 | 1 | 2 | 3 | 6 | 9 | 15 | 22 | 31 | 43 | 57 | 74 | 94 | 114 | 136 | 158 | 178 | 198 | 215 | 229 | 241 | 250 | 258 | 263 | 266 | 269 | 270 | 271 | 272 | 272 |
| -0.4 | 0 | 0 | 1 | 2 | 4 | 7 | 12 | 18 | 27 | 39 | 54 | 72 | 94 | 118 | 144 | 171 | 198 | 224 | 249 | 270 | 288 | 303 | 315 | 324 | 330 | 335 | 338 | 340 | 341 | 342 | 342 |
| -0.2 | 0 | 1 | 1 | 3 | 5 | 9 | 15 | 22 | 33 | 48 | 66 | 88 | 114 | 144 | 176 | 209 | 242 | 274 | 304 | 330 | 352 | 371 | 385 | 396 | 404 | 409 | 413 | 415 | 417 | 418 | 418 |
| 0.0 | 0 | 1 | 2 | 3 | 6 | 11 | 17 | 27 | 40 | 57 | 78 | 105 | 136 | 171 | 209 | 249 | 288 | 326 | 361 | 392 | 419 | 441 | 458 | 471 | 480 | 487 | 491 | 494 | 496 | 497 | 497 |
| 0.2 | 0 | 1 | 2 | 4 | 7 | 12 | 20 | 31 | 46 | 66 | 91 | 122 | 158 | 198 | 242 | 288 | 334 | 378 | 419 | 455 | 485 | 511 | 530 | 545 | 556 | 564 | 569 | 572 | 574 | 576 | 576 |
| 0.4 | 0 | 1 | 2 | 4 | 8 | 14 | 23 | 35 | 52 | 74 | 103 | 138 | 178 | 224 | 274 | 326 | 379 | 428 | 474 | 515 | 549 | 576 | 600 | 617 | 630 | 638 | 644 | 648 | 650 | 651 | 652 |
| 0.6 | 0 | 1 | 2 | 5 | 9 | 16 | 25 | 39 | 58 | 82 | 114 | 152 | 199 | 249 | 304 | 361 | 419 | 474 | 525 | 570 | 608 | 640 | 665 | 684 | 697 | 707 | 713 | 717 | 720 | 722 | 722 |
| 0.8 | 0 | 1 | 3 | 5 | 10 | 17 | 27 | 42 | 62 | 89 | 124 | 166 | 215 | 270 | 330 | 392 | 455 | 515 | 570 | 619 | 661 | 695 | 722 | 743 | 757 | 768 | 775 | 779 | 782 | 784 | 785 |
| 1.0 | 0 | 1 | 3 | 6 | 11 | 18 | 29 | 45 | 67 | 96 | 132 | 177 | 229 | 288 | 352 | 419 | 485 | 549 | 605 | 661 | 706 | 742 | 771 | 793 | 809 | 820 | 827 | 832 | 835 | 837 | 838 |
| 1.2 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 47 | 70 | 100 | 139 | 186 | 241 | 303 | 371 | 441 | 511 | 578 | 640 | 695 | 742 | 781 | 811 | 834 | 851 | 862 | 870 | 875 | 878 | 880 | 881 |
| 1.4 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 73 | 104 | 144 | 193 | 250 | 315 | 385 | 458 | 530 | 600 | 665 | 722 | 771 | 811 | 843 | 866 | 884 | 896 | 904 | 909 | 912 | 914 | 915 |
| 1.6 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 50 | 75 | 107 | 148 | 199 | 258 | 324 | 396 | 471 | 545 | 617 | 684 | 743 | 793 | 834 | 866 | 891 | 909 | 921 | 929 | 935 | 938 | 940 | 941 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 51 | 76 | 109 | 151 | 203 | 263 | 330 | 404 | 480 | 556 | 630 | 697 | 757 | 809 | 851 | 884 | 909 | 927 | 939 | 948 | 953 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 111 | 154 | 205 | 266 | 335 | 409 | 487 | 564 | 638 | 707 | 768 | 820 | 862 | 896 | 921 | 939 | 952 | 961 | 967 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 207 | 269 | 338 | 413 | 491 | 569 | 644 | 713 | 775 | 827 | 870 | 904 | 929 | 948 | 961 | 970 | 975 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 208 | 270 | 340 | 415 | 494 | 572 | 648 | 717 | 779 | 832 | 875 | 909 | 935 | 953 | 967 | 975 | 981 | 984 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 496 | 574 | 650 | 720 | 782 | 835 | 878 | 912 | 938 | 957 | 970 | 979 | 984 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 13 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 837 | 880 | 914 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 13 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 785 | 838 | 881 | 915 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
P: 0.1

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 13 | 13 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 | 16 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 23 | 25 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 8 | 11 | 14 | 18 | 22 | 27 | 31 | 35 | 39 | 42 | 45 | 47 | 49 | 50 | 51 | 52 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 0 | 1 | 2 | 3 | 4 | 6 | 9 | 12 | 16 | 21 | 27 | 33 | 39 | 45 | 51 | 57 | 62 | 66 | 70 | 73 | 75 | 76 | 77 | 78 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 12 | 17 | 23 | 30 | 37 | 46 | 55 | 64 | 73 | 81 | 88 | 95 | 100 | 104 | 107 | 109 | 111 | 112 | 112 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 11 | 15 | 23 | 31 | 40 | 51 | 63 | 75 | 88 | 100 | 111 | 122 | 130 | 137 | 143 | 148 | 151 | 153 | 155 | 156 | 156 | 157 | 157 | 157 |
| -0.8 | 0 | 0 | 1 | 2 | 4 | 6 | 10 | 14 | 21 | 30 | 40 | 53 | 67 | 83 | 99 | 116 | 133 | 148 | 162 | 173 | 183 | 191 | 197 | 202 | 205 | 207 | 208 | 209 | 209 | 210 | 210 |
| -0.6 | 0 | 0 | 1 | 2 | 5 | 8 | 12 | 18 | 27 | 37 | 51 | 67 | 85 | 106 | 127 | 149 | 170 | 190 | 208 | 224 | 237 | 247 | 255 | 261 | 265 | 268 | 270 | 271 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 1 | 3 | 5 | 9 | 15 | 22 | 33 | 46 | 63 | 83 | 106 | 131 | 158 | 186 | 212 | 238 | 260 | 280 | 297 | 310 | 320 | 328 | 333 | 337 | 339 | 341 | 341 | 342 | 342 |
| -0.2 | 0 | 1 | 2 | 4 | 6 | 11 | 17 | 27 | 39 | 55 | 75 | 99 | 127 | 158 | 191 | 224 | 257 | 288 | 316 | 341 | 361 | 378 | 390 | 400 | 407 | 411 | 414 | 416 | 417 | 418 | 418 |
| 0.0 | 0 | 1 | 2 | 4 | 7 | 13 | 20 | 31 | 45 | 64 | 88 | 116 | 149 | 186 | 224 | 264 | 303 | 341 | 374 | 404 | 428 | 448 | 463 | 475 | 483 | 489 | 492 | 495 | 496 | 497 | 497 |
| 0.2 | 0 | 1 | 2 | 5 | 8 | 14 | 23 | 35 | 51 | 73 | 100 | 133 | 170 | 212 | 257 | 303 | 349 | 392 | 431 | 466 | 495 | 518 | 536 | 550 | 559 | 566 | 570 | 573 | 575 | 576 | 576 |
| 0.4 | 0 | 1 | 2 | 5 | 9 | 16 | 25 | 39 | 57 | 81 | 111 | 148 | 190 | 238 | 288 | 341 | 392 | 441 | 486 | 525 | 558 | 585 | 606 | 621 | 633 | 640 | 645 | 649 | 650 | 652 | 652 |
| 0.6 | 0 | 1 | 3 | 6 | 10 | 17 | 27 | 42 | 62 | 88 | 122 | 162 | 208 | 260 | 316 | 374 | 431 | 486 | 536 | 580 | 617 | 647 | 670 | 687 | 700 | 709 | 714 | 718 | 720 | 722 | 722 |
| 0.8 | 0 | 1 | 3 | 6 | 11 | 18 | 29 | 45 | 66 | 95 | 130 | 173 | 224 | 280 | 341 | 404 | 466 | 525 | 580 | 627 | 668 | 701 | 727 | 746 | 760 | 769 | 776 | 780 | 782 | 784 | 785 |
| 1.0 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 47 | 70 | 100 | 137 | 183 | 237 | 297 | 361 | 428 | 495 | 558 | 617 | 668 | 712 | 747 | 775 | 796 | 811 | 821 | 828 | 832 | 835 | 837 | 838 |
| 1.2 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 73 | 104 | 143 | 191 | 247 | 310 | 378 | 448 | 518 | 585 | 647 | 701 | 747 | 785 | 814 | 836 | 852 | 863 | 871 | 876 | 878 | 880 | 881 |
| 1.4 | 0 | 1 | 3 | 7 | 12 | 20 | 33 | 50 | 75 | 107 | 148 | 197 | 255 | 320 | 390 | 463 | 536 | 606 | 670 | 727 | 775 | 814 | 845 | 868 | 885 | 897 | 904 | 909 | 913 | 914 | 915 |
| 1.6 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 51 | 76 | 109 | 151 | 202 | 261 | 328 | 400 | 475 | 550 | 621 | 687 | 746 | 796 | 836 | 868 | 892 | 910 | 922 | 930 | 935 | 938 | 940 | 941 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 111 | 153 | 205 | 265 | 333 | 407 | 483 | 559 | 613 | 700 | 760 | 811 | 852 | 885 | 910 | 927 | 940 | 948 | 954 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 207 | 268 | 337 | 411 | 487 | 566 | 640 | 709 | 769 | 821 | 863 | 897 | 922 | 940 | 953 | 961 | 967 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 112 | 156 | 208 | 270 | 339 | 414 | 492 | 570 | 645 | 714 | 776 | 828 | 871 | 904 | 930 | 948 | 961 | 970 | 975 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 416 | 495 | 573 | 649 | 718 | 780 | 832 | 876 | 909 | 935 | 954 | 967 | 975 | 981 | 984 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 209 | 272 | 341 | 417 | 496 | 575 | 650 | 720 | 782 | 835 | 878 | 913 | 938 | 957 | 970 | 979 | 984 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 11 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 817 | 880 | 914 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 11 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 785 | 838 | 881 | 915 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
$\rho = 0.2$

| $Z_1$ \ $Z_2$ | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 11 | 13 | 14 | 16 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 10 | 12 | 15 | 18 | 20 | 23 | 25 | 28 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 1 | 2 | 3 | 4 | 6 | 8 | 11 | 14 | 18 | 22 | 26 | 31 | 35 | 39 | 42 | 45 | 47 | 49 | 50 | 51 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 1 | 1 | 2 | 4 | 5 | 8 | 11 | 15 | 20 | 26 | 32 | 38 | 45 | 51 | 57 | 62 | 66 | 70 | 73 | 75 | 76 | 77 | 78 | 78 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 11 | 15 | 21 | 28 | 36 | 44 | 53 | 63 | 72 | 80 | 87 | 94 | 95 | 103 | 107 | 109 | 111 | 112 | 112 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 0 | 1 | 2 | 4 | 6 | 10 | 14 | 20 | 28 | 37 | 48 | 59 | 72 | 85 | 97 | 109 | 119 | 129 | 136 | 142 | 147 | 150 | 153 | 154 | 155 | 156 | 156 | 157 | 157 | 157 |
| -0.8 | 0 | 0 | 1 | 3 | 5 | 8 | 12 | 18 | 25 | 36 | 48 | 62 | 77 | 94 | 111 | 128 | 143 | 158 | 170 | 181 | 189 | 196 | 201 | 204 | 206 | 208 | 209 | 209 | 210 | 210 | 210 |
| -0.6 | 0 | 1 | 2 | 3 | 6 | 9 | 15 | 22 | 32 | 44 | 59 | 77 | 97 | 118 | 140 | 162 | 183 | 202 | 219 | 233 | 244 | 253 | 259 | 264 | 267 | 269 | 270 | 271 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 2 | 4 | 7 | 11 | 18 | 26 | 38 | [illegible] | 72 | 94 | 118 | 145 | 172 | 200 | 226 | 251 | 272 | 290 | 305 | 316 | 325 | 331 | 335 | 338 | 340 | 341 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 2 | 4 | 8 | 13 | 20 | 31 | 45 | 63 | 85 | 111 | 140 | 172 | 206 | 240 | 272 | 302 | 329 | 352 | 370 | 385 | 396 | 404 | 409 | 413 | 415 | 417 | 417 | 418 | 418 |
| 0.0 | 0 | 1 | 2 | 5 | 9 | 14 | 23 | 35 | 53 | 72 | 97 | 128 | 162 | 200 | 240 | 280 | 319 | 355 | 387 | 415 | 436 | 456 | 469 | 479 | 486 | 490 | 493 | 495 | 496 | 497 | 497 |
| 0.2 | 0 | 1 | 3 | 5 | 9 | 16 | 25 | 39 | 57 | 80 | 109 | 141 | 183 | 226 | 272 | 319 | 364 | 407 | 445 | 477 | 504 | 526 | 542 | 559 | 562 | 568 | 572 | 574 | 575 | 576 | 576 |
| 0.4 | 0 | 1 | 3 | 6 | 10 | 17 | 28 | 42 | 62 | 87 | 119 | 158 | 202 | 251 | 302 | 355 | 407 | 455 | 499 | 536 | 568 | 593 | 612 | 626 | 636 | 642 | 647 | 649 | 651 | 652 | 652 |
| 0.6 | 0 | 1 | 3 | 6 | 11 | 18 | 29 | 45 | 66 | 94 | 129 | 170 | 219 | 272 | 329 | 387 | 445 | 499 | 547 | 590 | 625 | 654 | 675 | 691 | 703 | 711 | 716 | 719 | 721 | 722 | 723 |
| 0.8 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 47 | 70 | 99 | 136 | 181 | 233 | 290 | 352 | 415 | 477 | 536 | 590 | 637 | 676 | 707 | 732 | 750 | 762 | 771 | 777 | 781 | 783 | 784 | 785 |
| 1.0 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 73 | 103 | 142 | 189 | 244 | 305 | 370 | 438 | 504 | 568 | 625 | 676 | 718 | 752 | 779 | 799 | 813 | 823 | 829 | 833 | 836 | 837 | 838 |
| 1.2 | 0 | 1 | 3 | 7 | 12 | 20 | 33 | 50 | 75 | 107 | 147 | 196 | 253 | 316 | 385 | 456 | 526 | 593 | 654 | 707 | 752 | 789 | 818 | 839 | 854 | 865 | 872 | 876 | 879 | 880 | 881 |
| 1.4 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 51 | 76 | 109 | 150 | 201 | 259 | 325 | 396 | 469 | 542 | 612 | 675 | 732 | 779 | 818 | 848 | 870 | 887 | 898 | 905 | 910 | 913 | 915 | 916 |
| 1.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 111 | 153 | 204 | 264 | 331 | 404 | 479 | 554 | 626 | 691 | 750 | 799 | 839 | 870 | 894 | 911 | 923 | 930 | 935 | 939 | 940 | 941 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 154 | 206 | 267 | 315 | 409 | 486 | 562 | 636 | 703 | 762 | 813 | 854 | 887 | 911 | 928 | 941 | 949 | 954 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 76 | 112 | 155 | 208 | 269 | 338 | 413 | 490 | 568 | 642 | 711 | 771 | 823 | 865 | 898 | 923 | 941 | 953 | 962 | 967 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 270 | 340 | 415 | 493 | 572 | 647 | 716 | 777 | 829 | 872 | 905 | 930 | 949 | 962 | 970 | 976 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 495 | 574 | 649 | 719 | 781 | 833 | 876 | 910 | 935 | 954 | 967 | 976 | 981 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 417 | 496 | 575 | 651 | 721 | 783 | 836 | 879 | 913 | 939 | 957 | 970 | 979 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 837 | 880 | 915 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 723 | 785 | 838 | 881 | 916 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
P- 0.3

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 4 | 5 | 6 | 8 | 10 | 11 | 13 | 15 | 16 | 17 | 19 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 1 | 2 | 2 | 4 | 5 | 7 | 9 | 12 | 15 | 18 | 21 | 23 | 26 | 28 | 30 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 1 | 1 | 2 | 4 | 5 | 8 | 10 | 14 | 18 | 22 | 26 | 31 | 35 | 39 | 42 | 45 | 47 | 49 | 51 | 51 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 10 | 14 | 19 | 25 | 31 | 38 | 44 | 51 | 56 | 62 | 66 | 70 | 73 | 75 | 76 | 77 | 78 | 78 | 79 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 0 | 1 | 2 | 4 | 6 | 9 | 14 | 19 | 26 | 34 | 42 | 52 | 61 | 70 | 79 | 87 | 93 | 99 | 103 | 106 | 109 | 110 | 112 | 112 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 1 | 3 | 5 | 8 | 12 | 18 | 25 | 34 | 44 | 56 | 68 | 81 | 94 | 106 | 118 | 127 | 135 | 142 | 146 | 150 | 152 | 154 | 155 | 156 | 156 | 156 | 157 | 157 | 157 |
| -0.8 | 0 | 1 | 2 | 3 | 6 | 10 | 15 | 22 | 31 | 42 | 56 | 71 | 88 | 105 | 122 | 139 | 154 | 167 | 179 | 188 | 195 | 200 | 203 | 206 | 207 | 208 | 209 | 209 | 210 | 210 | 210 |
| -0.6 | 0 | 1 | 2 | 4 | 7 | 11 | 18 | 26 | 38 | 52 | 68 | 88 | 109 | 131 | 154 | 175 | 196 | 213 | 229 | 241 | 250 | 258 | 263 | 266 | 269 | 270 | 271 | 271 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 2 | 4 | 8 | 13 | 21 | 31 | 44 | 61 | 81 | 105 | 131 | 159 | 187 | 215 | 241 | 264 | 284 | 300 | 313 | 322 | 329 | 334 | 337 | 339 | 341 | 341 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 2 | 5 | 9 | 15 | 23 | 35 | 51 | 70 | 94 | 122 | 154 | 187 | 222 | 256 | 288 | 317 | 342 | 363 | 379 | 392 | 401 | 407 | 412 | 414 | 416 | 417 | 418 | 418 | 418 |
| 0.0 | 0 | 1 | 3 | 5 | 10 | 16 | 26 | 39 | 56 | 79 | 106 | 139 | 175 | 215 | 256 | 296 | 335 | 370 | 401 | 426 | 447 | 463 | 475 | 483 | 489 | 492 | 495 | 496 | 497 | 497 | 497 |
| 0.2 | 0 | 1 | 3 | 6 | 10 | 17 | 28 | 42 | 62 | 87 | 118 | 154 | 196 | 241 | 288 | 335 | 380 | 422 | 458 | 489 | 514 | 534 | 548 | 558 | 565 | 570 | 573 | 575 | 576 | 576 | 576 |
| 0.4 | 0 | 1 | 3 | 6 | 11 | 19 | 30 | 45 | 66 | 93 | 127 | 167 | 213 | 264 | 317 | 370 | 422 | 469 | 512 | 548 | 577 | 600 | 618 | 630 | 639 | 644 | 648 | 650 | 651 | 652 | 652 |
| 0.6 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 47 | 70 | 99 | 135 | 179 | 229 | 284 | 342 | 401 | 458 | 512 | 560 | 601 | 634 | 661 | 681 | 696 | 706 | 713 | 717 | 720 | 721 | 722 | 723 |
| 0.8 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 73 | 103 | 142 | 188 | 241 | 300 | 363 | 426 | 489 | 548 | 601 | 646 | 684 | 714 | 737 | 754 | 765 | 773 | 778 | 781 | 783 | 784 | 785 |
| 1.0 | 0 | 1 | 3 | 7 | 12 | 20 | 33 | 51 | 75 | 106 | 146 | 195 | 250 | 313 | 379 | 447 | 514 | 577 | 634 | 684 | 726 | 759 | 784 | 803 | 816 | 825 | 830 | 834 | 836 | 837 | 838 |
| 1.2 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 51 | 76 | 109 | 150 | 200 | 258 | 322 | 392 | 463 | 534 | 600 | 661 | 714 | 759 | 794 | 822 | 842 | 857 | 866 | 873 | 877 | 879 | 881 | 881 |
| 1.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 110 | 152 | 203 | 263 | 329 | 401 | 475 | 548 | 618 | 681 | 737 | 784 | 822 | 851 | 873 | 889 | 899 | 906 | 911 | 913 | 915 | 916 |
| 1.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 154 | 206 | 266 | 334 | 407 | 483 | 558 | 630 | 696 | 754 | 803 | 842 | 873 | 896 | 912 | 924 | 931 | 935 | 939 | 941 | 941 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 207 | 269 | 337 | 412 | 489 | 565 | 639 | 706 | 765 | 816 | 857 | 889 | 912 | 930 | 942 | 949 | 954 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 208 | 270 | 339 | 414 | 492 | 570 | 644 | 713 | 773 | 825 | 866 | 899 | 924 | 942 | 954 | 962 | 967 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 416 | 495 | 573 | 648 | 717 | 778 | 830 | 873 | 906 | 931 | 949 | 962 | 970 | 976 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 496 | 575 | 650 | 720 | 781 | 834 | 877 | 911 | 936 | 954 | 967 | 976 | 981 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 651 | 721 | 783 | 836 | 879 | 913 | 939 | 957 | 970 | 979 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 837 | 881 | 915 | 941 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 723 | 785 | 818 | 881 | 916 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
P=0.4000

| | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 1 | 2 | 2 | 3 | 5 | 6 | 8 | 10 | 12 | 13 | 15 | 17 | 19 | 19 | 20 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 7 | 9 | 12 | 15 | 18 | 21 | 24 | 26 | 28 | 30 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 10 | 13 | 17 | 22 | 26 | 31 | 35 | 39 | 43 | 45 | 48 | 49 | 51 | 52 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 1 | 2 | 4 | 6 | 9 | 13 | 18 | 24 | 30 | 37 | 44 | 50 | 56 | 62 | 66 | 70 | 73 | 75 | 76 | 77 | 78 | 78 | 79 | 79 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 1 | 1 | 3 | 5 | 8 | 12 | 17 | 24 | 32 | 40 | 50 | 59 | 69 | 78 | 86 | 93 | 99 | 103 | 107 | 109 | 111 | 112 | 112 | 113 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 2 | 3 | 6 | 10 | 15 | 22 | 30 | 40 | 52 | 65 | 78 | 91 | 104 | 116 | 126 | 134 | 141 | 146 | 150 | 152 | 154 | 155 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| -0.8 | 0 | 1 | 2 | 4 | 7 | 12 | 18 | 26 | 37 | 50 | 65 | 81 | 99 | 117 | 134 | 150 | 165 | 177 | 186 | 194 | 199 | 203 | 206 | 207 | 208 | 209 | 209 | 209 | 210 | 210 | 210 |
| -0.6 | 0 | 1 | 2 | 5 | 8 | 13 | 21 | 31 | 44 | 59 | 78 | 99 | 122 | 145 | 168 | 189 | 208 | 225 | 238 | 249 | 256 | 262 | 266 | 268 | 270 | 271 | 271 | 272 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 3 | 5 | 9 | 15 | 24 | 35 | 50 | 69 | 91 | 117 | 145 | 174 | 203 | 230 | 255 | 277 | 295 | 309 | 320 | 328 | 333 | 337 | 339 | 340 | 341 | 342 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 3 | 6 | 10 | 17 | 26 | 39 | 56 | 78 | 104 | 134 | 168 | 203 | 238 | 272 | 304 | 331 | 354 | 373 | 388 | 398 | 406 | 411 | 414 | 416 | 417 | 418 | 418 | 418 | 418 |
| 0.0 | 0 | 1 | 3 | 6 | 11 | 18 | 28 | 43 | 62 | 86 | 116 | 150 | 189 | 230 | 272 | 313 | 351 | 385 | 414 | 438 | 457 | 470 | 480 | 487 | 491 | 494 | 496 | 496 | 497 | 497 | 497 |
| 0.2 | 0 | 1 | 3 | 6 | 11 | 19 | 30 | 45 | 66 | 93 | 126 | 165 | 208 | 255 | 304 | 351 | 396 | 437 | 472 | 501 | 524 | 542 | 554 | 563 | 568 | 572 | 574 | 575 | 576 | 576 | 577 |
| 0.4 | 0 | 1 | 3 | 6 | 12 | 20 | 31 | 48 | 70 | 99 | 134 | 177 | 225 | 277 | 331 | 385 | 437 | 484 | 526 | 560 | 588 | 609 | 624 | 635 | 642 | 646 | 649 | 651 | 652 | 652 | 653 |
| 0.6 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 73 | 103 | 141 | 186 | 238 | 295 | 354 | 414 | 472 | 526 | 573 | 612 | 644 | 669 | 688 | 701 | 709 | 715 | 719 | 721 | 722 | 722 | 723 |
| 0.8 | 0 | 1 | 3 | 7 | 12 | 20 | 33 | 51 | 75 | 107 | 146 | 194 | 249 | 309 | 373 | 433 | 501 | 560 | 612 | 657 | 693 | 722 | 741 | 758 | 769 | 776 | 780 | 782 | 784 | 785 | 785 |
| 1.0 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 52 | 76 | 109 | 150 | 199 | 256 | 320 | 388 | 457 | 524 | 588 | 644 | 693 | 734 | 765 | 789 | 807 | 819 | 827 | 832 | 835 | 837 | 838 | 838 |
| 1.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 111 | 152 | 203 | 262 | 328 | 398 | 470 | 542 | 609 | 669 | 722 | 765 | 800 | 827 | 846 | 859 | 868 | 874 | 878 | 880 | 881 | 882 |
| 1.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 154 | 206 | 266 | 333 | 406 | 480 | 554 | 624 | 688 | 743 | 789 | 827 | 855 | 876 | 891 | 901 | 907 | 911 | 914 | 915 | 916 |
| 1.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 207 | 268 | 337 | 411 | 487 | 563 | 635 | 701 | 758 | 807 | 846 | 876 | 899 | 914 | 925 | 932 | 937 | 939 | 941 | 942 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 208 | 270 | 339 | 414 | 491 | 568 | 642 | 709 | 769 | 819 | 859 | 891 | 914 | 931 | 943 | 950 | 955 | 958 | 960 | 960 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 340 | 416 | 494 | 572 | 647 | 715 | 776 | 827 | 868 | 901 | 925 | 943 | 955 | 963 | 968 | 971 | 973 | 974 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 496 | 574 | 649 | 719 | 780 | 832 | 874 | 907 | 932 | 950 | 963 | 971 | 976 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 496 | 575 | 651 | 721 | 782 | 835 | 878 | 911 | 937 | 955 | 968 | 976 | 982 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 837 | 880 | 914 | 939 | 958 | 971 | 979 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 785 | 838 | 881 | 915 | 941 | 960 | 973 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 210 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 882 | 916 | 942 | 960 | 974 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES

P = 0.5

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 12 | 14 | 15 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 9 | 12 | 15 | 18 | 21 | 24 | 26 | 29 | 30 | 32 | 32 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 1 | 2 | 4 | 6 | 9 | 11 | 17 | 21 | 26 | 31 | 35 | 40 | 43 | 46 | 48 | 50 | 51 | 52 | 52 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 1 | 2 | 3 | 5 | 8 | 12 | 17 | 22 | 29 | 36 | 43 | 50 | 56 | 62 | 67 | 70 | 73 | 75 | 77 | 77 | 78 | 78 | 78 | 79 | 79 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 1 | 2 | 4 | 6 | 10 | 15 | 21 | 29 | 38 | 48 | 58 | 68 | 77 | 86 | 93 | 99 | 104 | 107 | 109 | 111 | 112 | 112 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 2 | 4 | 7 | 12 | 18 | 26 | 36 | 48 | 61 | 74 | 88 | 102 | 114 | 125 | 134 | 141 | 146 | 150 | 153 | 154 | 155 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| -0.8 | 0 | 1 | 2 | 5 | 8 | 14 | 21 | 31 | 43 | 58 | 74 | 93 | 111 | 130 | 147 | 162 | 175 | 185 | 193 | 199 | 203 | 206 | 207 | 208 | 209 | 209 | 209 | 209 | 209 | 209 | 209 |
| -0.6 | 0 | 1 | 3 | 5 | 9 | 15 | 24 | 35 | 50 | 68 | 88 | 111 | 135 | 159 | 182 | 203 | 221 | 236 | 247 | 256 | 262 | 266 | 268 | 270 | 271 | 271 | 272 | 272 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 3 | 6 | 10 | 17 | 26 | 40 | 56 | 77 | 102 | 130 | 159 | 190 | 219 | 246 | 270 | 290 | 306 | 318 | 327 | 333 | 337 | 339 | 340 | 341 | 342 | 342 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 3 | 6 | 11 | 18 | 29 | 43 | 62 | 86 | 114 | 147 | 182 | 219 | 255 | 289 | 320 | 346 | 367 | 384 | 396 | 404 | 410 | 413 | 416 | 417 | 418 | 418 | 418 | 418 | 418 |
| 0.0 | 0 | 1 | 3 | 6 | 11 | 19 | 30 | 46 | 67 | 93 | 125 | 162 | 203 | 246 | 289 | 331 | 369 | 401 | 429 | 450 | 466 | 478 | 485 | 490 | 494 | 495 | 496 | 497 | 497 | 497 | 497 |
| 0.2 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 48 | 70 | 99 | 134 | 175 | 221 | 270 | 320 | 369 | 414 | 454 | 487 | 514 | 535 | 550 | 560 | 567 | 571 | 574 | 575 | 576 | 576 | 576 | 577 |
| 0.4 | 0 | 1 | 3 | 7 | 12 | 20 | 32 | 50 | 73 | 104 | 141 | 185 | 236 | 290 | 346 | 401 | 454 | 500 | 540 | 573 | 598 | 617 | 630 | 639 | 645 | 649 | 651 | 652 | 652 | 653 | 653 |
| 0.6 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 51 | 75 | 107 | 146 | 193 | 247 | 306 | 367 | 429 | 487 | 540 | 586 | 625 | 655 | 678 | 694 | 706 | 713 | 717 | 720 | 722 | 722 | 723 | 723 |
| 0.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 109 | 150 | 199 | 256 | 318 | 384 | 450 | 514 | 573 | 625 | 668 | 703 | 730 | 750 | 763 | 772 | 778 | 781 | 783 | 784 | 785 | 785 |
| 1.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 111 | 153 | 203 | 262 | 327 | 396 | 466 | 535 | 598 | 655 | 703 | 743 | 773 | 796 | 812 | 822 | 829 | 833 | 836 | 837 | 838 | 838 |
| 1.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 154 | 206 | 266 | 333 | 404 | 478 | 550 | 617 | 678 | 730 | 773 | 807 | 832 | 850 | 863 | 871 | 876 | 879 | 880 | 881 | 882 |
| 1.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 207 | 268 | 337 | 410 | 485 | 560 | 630 | 694 | 750 | 796 | 832 | 860 | 880 | 894 | 903 | 909 | 912 | 914 | 915 | 916 |
| 1.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 208 | 270 | 339 | 414 | 490 | 567 | 639 | 706 | 763 | 812 | 850 | 880 | 902 | 917 | 927 | 934 | 938 | 940 | 941 | 942 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 340 | 416 | 494 | 571 | 645 | 713 | 772 | 822 | 863 | 894 | 917 | 933 | 944 | 951 | 956 | 958 | 960 | 961 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 495 | 574 | 649 | 717 | 778 | 829 | 871 | 903 | 927 | 944 | 956 | 964 | 968 | 971 | 973 | 974 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 496 | 575 | 651 | 720 | 781 | 833 | 876 | 909 | 934 | 951 | 964 | 972 | 977 | 980 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 783 | 836 | 879 | 912 | 938 | 956 | 968 | 977 | 982 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 837 | 880 | 914 | 940 | 958 | 971 | 980 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 576 | 653 | 723 | 785 | 838 | 881 | 915 | 941 | 960 | 973 | 981 | 987 | 990 | 992 | 994 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 882 | 916 | 942 | 961 | 974 | 982 | 988 | 992 | 994 | 995 |

BIVARIATE NORMAL TABLES
P- 0.600

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 1 | 2 | 4 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 1 | 2 | 3 | 5 | 8 | 12 | 16 | 21 | 26 | 31 | 36 | 40 | 44 | 47 | 49 | 50 | 51 | 52 | 52 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 1 | 2 | 4 | 6 | 10 | 15 | 21 | 27 | 35 | 42 | 50 | 57 | 63 | 68 | 71 | 74 | 76 | 77 | 78 | 78 | 78 | 78 | 78 | 79 | 79 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 1 | 2 | 4 | 8 | 12 | 18 | 26 | 35 | 45 | 56 | 67 | 77 | 86 | 94 | 100 | 104 | 108 | 110 | 111 | 112 | 112 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 2 | 5 | 9 | 14 | 21 | 31 | 42 | 56 | 70 | 85 | 100 | 113 | 125 | 134 | 142 | 147 | 151 | 153 | 155 | 155 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| -0.8 | 0 | 1 | 3 | 5 | 10 | 16 | 24 | 36 | 50 | 67 | 85 | 105 | 125 | 143 | 160 | 174 | 185 | 194 | 200 | 204 | 206 | 208 | 209 | 209 | 209 | 209 | 209 | 209 | 209 | 209 | 209 |
| -0.6 | 0 | 1 | 3 | 6 | 10 | 17 | 27 | 40 | 57 | 77 | 100 | 125 | 150 | 175 | 198 | 218 | 234 | 247 | 256 | 262 | 266 | 269 | 270 | 271 | 271 | 272 | 272 | 272 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 3 | 6 | 11 | 18 | 29 | 44 | 63 | 86 | 111 | 143 | 175 | 207 | 237 | 263 | 286 | 303 | 317 | 326 | 333 | 337 | 339 | 341 | 341 | 342 | 342 | 342 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 47 | 68 | 94 | 125 | 160 | 198 | 237 | 274 | 308 | 337 | 362 | 380 | 394 | 404 | 410 | 414 | 416 | 417 | 418 | 418 | 418 | 418 | 418 | 418 |
| 0.0 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 71 | 100 | 134 | 174 | 218 | 263 | 308 | 350 | 387 | 418 | 443 | 462 | 475 | 484 | 490 | 494 | 495 | 496 | 497 | 497 | 497 | 497 | 497 |
| 0.2 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 50 | 74 | 104 | 142 | 185 | 234 | 286 | 337 | 387 | 432 | 471 | 503 | 527 | 545 | 558 | 566 | 571 | 574 | 575 | 576 | 576 | 577 | 577 | 577 |
| 0.4 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 51 | 76 | 108 | 147 | 194 | 247 | 303 | 362 | 418 | 471 | 517 | 556 | 587 | 610 | 626 | 637 | 644 | 648 | 650 | 652 | 652 | 653 | 653 | 653 |
| 0.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 77 | 110 | 151 | 200 | 256 | 317 | 380 | 443 | 503 | 556 | 602 | 638 | 667 | 687 | 701 | 711 | 716 | 720 | 721 | 722 | 723 | 723 | 723 |
| 0.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 111 | 153 | 204 | 262 | 326 | 394 | 462 | 527 | 587 | 638 | 681 | 714 | 739 | 757 | 769 | 776 | 780 | 783 | 784 | 785 | 785 | 785 |
| 1.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 155 | 206 | 266 | 333 | 404 | 475 | 545 | 610 | 667 | 714 | 753 | 782 | 803 | 817 | 826 | 832 | 835 | 837 | 838 | 838 | 839 |
| 1.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 155 | 208 | 269 | 337 | 410 | 484 | 558 | 626 | 687 | 739 | 782 | 814 | 838 | 855 | 866 | 873 | 878 | 880 | 881 | 882 | 882 |
| 1.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 270 | 339 | 414 | 490 | 566 | 637 | 701 | 757 | 803 | 838 | 865 | 884 | 897 | 906 | 911 | 913 | 915 | 916 | 916 |
| 1.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 271 | 341 | 416 | 494 | 571 | 644 | 711 | 769 | 817 | 855 | 884 | 905 | 920 | 929 | 935 | 939 | 941 | 942 | 942 |
| 1.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 495 | 574 | 648 | 716 | 776 | 826 | 866 | 897 | 920 | 936 | 946 | 953 | 957 | 959 | 960 | 961 |
| 2.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 496 | 575 | 650 | 720 | 780 | 832 | 873 | 906 | 929 | 946 | 957 | 965 | 969 | 972 | 973 | 974 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 576 | 652 | 721 | 783 | 835 | 878 | 911 | 935 | 953 | 965 | 972 | 977 | 980 | 982 | 983 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 576 | 652 | 722 | 784 | 837 | 880 | 913 | 939 | 957 | 969 | 977 | 982 | 986 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 881 | 915 | 941 | 959 | 972 | 980 | 986 | 989 | 991 | 992 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 882 | 916 | 942 | 960 | 973 | 982 | 987 | 991 | 993 | 994 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 839 | 882 | 916 | 942 | 961 | 974 | 983 | 988 | 992 | 994 | 995 |

BIVARIATE NORMAL TABLES
ρ 0.7

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 1 |
| -2.4 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| -2.2 | 0 | 0 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 1 | 1 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 16 | 18 | 19 | 20 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 1 | 2 | 3 | 5 | 8 | 11 | 15 | 18 | 22 | 25 | 28 | 30 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 1 | 2 | 4 | 7 | 10 | 15 | 20 | 26 | 31 | 37 | 41 | 45 | 48 | 50 | 51 | 52 | 52 | 52 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 1 | 2 | 5 | 8 | 13 | 18 | 26 | 34 | 42 | 50 | 58 | 64 | 69 | 72 | 75 | 77 | 77 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 70 | 78 | 78 | 78 | 78 | 78 |
| -1.2 | 0 | 1 | 3 | 5 | 9 | 15 | 22 | 31 | 42 | 54 | 65 | 77 | 87 | 95 | 101 | 106 | 109 | 111 | 112 | 112 | 112 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 3 | 6 | 10 | 16 | 25 | 37 | 50 | 65 | 82 | 98 | 112 | 125 | 135 | 143 | 148 | 152 | 154 | 155 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| -0.8 | 0 | 1 | 3 | 6 | 11 | 18 | 28 | 41 | 58 | 77 | 98 | 119 | 140 | 158 | 174 | 186 | 195 | 201 | 205 | 207 | 208 | 209 | 209 | 209 | 209 | 209 | 209 | 209 | 209 | 209 | 209 |
| -0.6 | 0 | 1 | 3 | 6 | 11 | 19 | 30 | 45 | 64 | 87 | 112 | 140 | 167 | 193 | 215 | 233 | 247 | 257 | 263 | 267 | 269 | 271 | 271 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 3 | 6 | 12 | 20 | 31 | 48 | 69 | 95 | 125 | 158 | 193 | 226 | 256 | 281 | 302 | 317 | 327 | 334 | 338 | 340 | 341 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 50 | 72 | 101 | 135 | 174 | 215 | 256 | 294 | 328 | 356 | 378 | 393 | 404 | 410 | 414 | 416 | 417 | 418 | 418 | 418 | 418 | 418 | 418 | 418 |
| 0.0 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 51 | 75 | 106 | 143 | 186 | 233 | 281 | 328 | 371 | 407 | 437 | 459 | 474 | 484 | 491 | 494 | 496 | 497 | 497 | 497 | 497 | 498 | 498 | 498 |
| 0.2 | 0 | 1 | 3 | 6 | 12 | 21 | 33 | 52 | 77 | 109 | 148 | 195 | 247 | 302 | 356 | 407 | 453 | 491 | 520 | 541 | 556 | 565 | 571 | 574 | 575 | 576 | 577 | 577 | 577 | 577 | 577 |
| 0.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 77 | 111 | 152 | 201 | 257 | 317 | 378 | 437 | 491 | 537 | 574 | 602 | 622 | 635 | 643 | 648 | 651 | 652 | 653 | 653 | 653 | 653 | 653 |
| 0.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 78 | 112 | 154 | 205 | 263 | 327 | 393 | 459 | 520 | 574 | 619 | 654 | 679 | 697 | 709 | 716 | 719 | 721 | 722 | 723 | 723 | 723 | 723 |
| 0.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 78 | 112 | 155 | 207 | 267 | 334 | 404 | 474 | 541 | 602 | 654 | 695 | 727 | 750 | 765 | 775 | 780 | 783 | 784 | 785 | 785 | 786 | 786 |
| 1.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 112 | 156 | 208 | 270 | 338 | 410 | 484 | 556 | 622 | 679 | 727 | 764 | 792 | 810 | 823 | 830 | 835 | 837 | 838 | 838 | 839 | 839 |
| 1.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 271 | 340 | 414 | 491 | 565 | 635 | 697 | 759 | 792 | 823 | 846 | 861 | 871 | 876 | 879 | 881 | 882 | 882 | 882 |
| 1.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 271 | 341 | 416 | 494 | 571 | 643 | 709 | 765 | 810 | 846 | 872 | 890 | 901 | 908 | 913 | 915 | 916 | 916 | 917 |
| 1.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 417 | 496 | 574 | 648 | 716 | 774 | 823 | 861 | 890 | 910 | 923 | 932 | 937 | 940 | 941 | 942 | 942 |
| 1.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 575 | 651 | 719 | 780 | 830 | 871 | 901 | 923 | 939 | 948 | 954 | 958 | 960 | 961 | 961 |
| 2.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 576 | 652 | 721 | 783 | 835 | 876 | 908 | 932 | 948 | 959 | 966 | 973 | 972 | 974 | 974 |
| 2.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 722 | 784 | 837 | 879 | 913 | 937 | 954 | 966 | 974 | 978 | 981 | 982 | 983 |
| 2.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 881 | 915 | 940 | 958 | 970 | 978 | 983 | 986 | 988 | 989 |
| 2.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 785 | 838 | 882 | 916 | 941 | 960 | 972 | 981 | 986 | 989 | 991 | 992 |
| 2.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 882 | 916 | 942 | 961 | 974 | 982 | 988 | 991 | 993 | 994 |
| 3.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 882 | 917 | 942 | 961 | 974 | 983 | 989 | 992 | 994 | 995 |

BIVARIATE NORMAL TABLES
ρ= 0.8

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| -2.2 | 0 | 1 | 1 | 2 | 4 | 5 | 7 | 8 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 1 | 2 | 3 | 5 | 8 | 11 | 13 | 16 | 17 | 19 | 20 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 1 | 2 | 4 | 7 | 11 | 15 | 19 | 23 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 1 | 2 | 5 | 8 | 13 | 19 | 25 | 32 | 38 | 43 | 47 | 49 | 51 | 52 | 52 | 52 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 1 | 3 | 5 | 10 | 16 | 23 | 32 | 41 | 51 | 59 | 66 | 71 | 74 | 76 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 |
| -1.2 | 0 | 1 | 3 | 6 | 11 | 17 | 27 | 38 | 51 | 64 | 77 | 88 | 97 | 104 | 108 | 110 | 112 | 112 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 3 | 6 | 11 | 19 | 29 | 43 | 59 | 77 | 95 | 112 | 127 | 138 | 146 | 151 | 154 | 155 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 | 156 |
| -0.8 | 0 | 1 | 3 | 6 | 12 | 20 | 31 | 47 | 66 | 88 | 112 | 136 | 157 | 175 | 189 | 198 | 203 | 207 | 208 | 209 | 209 | 209 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 |
| -0.6 | 0 | 1 | 3 | 6 | 12 | 20 | 32 | 49 | 71 | 97 | 127 | 157 | 187 | 213 | 234 | 249 | 259 | 266 | 269 | 271 | 271 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 51 | 74 | 104 | 138 | 175 | 213 | 248 | 278 | 302 | 318 | 329 | 336 | 339 | 341 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 3 | 6 | 12 | 21 | 33 | 52 | 76 | 108 | 146 | 189 | 234 | 278 | 318 | 351 | 377 | 395 | 406 | 412 | 416 | 417 | 418 | 418 | 418 | 418 | 418 | 418 | 418 | 418 | 418 |
| 0.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 78 | 110 | 151 | 198 | 249 | 302 | 351 | 395 | 431 | 457 | 475 | 486 | 492 | 495 | 497 | 497 | 498 | 498 | 498 | 498 | 498 | 498 | 498 |
| 0.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 78 | 112 | 154 | 203 | 259 | 318 | 377 | 431 | 477 | 513 | 539 | 556 | 566 | 572 | 575 | 576 | 577 | 577 | 577 | 577 | 577 | 577 | 577 |
| 0.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 78 | 112 | 155 | 207 | 266 | 329 | 395 | 457 | 513 | 559 | 594 | 619 | 635 | 644 | 649 | 651 | 652 | 653 | 653 | 653 | 653 | 653 | 653 |
| 0.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 208 | 269 | 336 | 406 | 475 | 539 | 594 | 638 | 671 | 694 | 708 | 716 | 720 | 722 | 723 | 723 | 723 | 723 | 723 | 723 |
| 0.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 271 | 339 | 412 | 486 | 556 | 619 | 671 | 712 | 742 | 761 | 773 | 780 | 783 | 785 | 785 | 786 | 786 | 786 | 786 |
| 1.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 271 | 341 | 416 | 492 | 566 | 635 | 694 | 742 | 778 | 803 | 820 | 829 | 835 | 837 | 838 | 839 | 839 | 839 | 839 |
| 1.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 417 | 495 | 572 | 644 | 708 | 761 | 803 | 834 | 855 | 868 | 875 | 879 | 881 | 882 | 882 | 883 | 883 |
| 1.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 497 | 575 | 649 | 716 | 773 | 820 | 855 | 880 | 896 | 906 | 912 | 915 | 916 | 917 | 917 | 917 |
| 1.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 497 | 576 | 651 | 720 | 780 | 829 | 868 | 896 | 916 | 928 | 935 | 939 | 941 | 942 | 943 | 943 |
| 1.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 652 | 722 | 783 | 835 | 875 | 906 | 928 | 943 | 952 | 957 | 959 | 961 | 961 | 962 |
| 2.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 785 | 837 | 879 | 912 | 935 | 952 | 962 | 968 | 972 | 973 | 974 | 975 |
| 2.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 785 | 838 | 881 | 915 | 939 | 957 | 968 | 975 | 980 | 982 | 983 | 983 |
| 2.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 882 | 916 | 941 | 959 | 972 | 980 | 984 | 987 | 988 | 989 |
| 2.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 882 | 917 | 942 | 961 | 973 | 982 | 987 | 990 | 991 | 992 |
| 2.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 883 | 917 | 943 | 961 | 974 | 983 | 988 | 991 | 993 | 994 |
| 3.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 883 | 917 | 943 | 962 | 975 | 983 | 989 | 992 | 994 | 995 |

BIVARIATE NORMAL TABLES
ρ = 0.9

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 1 | 1 | 3 | 4 | 5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| -2.2 | 0 | 1 | 2 | 4 | 6 | 8 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 1 | 2 | 5 | 8 | 11 | 15 | 17 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 1 | 3 | 6 | 10 | 15 | 20 | 25 | 29 | 31 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 1 | 3 | 6 | 11 | 17 | 25 | 33 | 40 | 46 | 50 | 51 | 52 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| -1.4 | 0 | 1 | 3 | 6 | 11 | 19 | 29 | 40 | 52 | 62 | 70 | 75 | 77 | 78 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 |
| -1.2 | 0 | 1 | 3 | 6 | 12 | 20 | 31 | 46 | 62 | 78 | 92 | 102 | 108 | 111 | 112 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 | 113 |
| -1.0 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 50 | 70 | 92 | 113 | 131 | 143 | 151 | 154 | 156 | 156 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 | 157 |
| -0.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 51 | 75 | 102 | 131 | 158 | 180 | 194 | 203 | 207 | 209 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 |
| -0.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 77 | 108 | 143 | 180 | 212 | 238 | 255 | 265 | 270 | 271 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 | 272 |
| -0.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 111 | 151 | 194 | 238 | 276 | 305 | 324 | 335 | 340 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 | 342 |
| -0.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 112 | 154 | 203 | 255 | 305 | 348 | 380 | 400 | 411 | 416 | 418 | 418 | 419 | 419 | 419 | 419 | 419 | 419 | 419 | 419 | 419 | 419 |
| 0.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 207 | 265 | 324 | 380 | 426 | 459 | 480 | 491 | 495 | 497 | 498 | 498 | 498 | 498 | 498 | 498 | 498 | 498 | 498 | 498 |
| 0.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 270 | 335 | 400 | 459 | 507 | 540 | 560 | 571 | 575 | 577 | 577 | 577 | 577 | 577 | 577 | 577 | 577 | 577 | 577 |
| 0.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 271 | 340 | 411 | 480 | 540 | 587 | 619 | 638 | 647 | 651 | 653 | 653 | 653 | 653 | 653 | 653 | 653 | 653 | 653 |
| 0.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 416 | 491 | 560 | 619 | 664 | 698 | 710 | 719 | 722 | 723 | 724 | 724 | 724 | 724 | 724 | 724 | 724 |
| 0.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 495 | 571 | 638 | 696 | 734 | 761 | 775 | 782 | 785 | 786 | 786 | 786 | 786 | 786 | 786 | 786 |
| 1.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 575 | 647 | 710 | 761 | 796 | 818 | 830 | 836 | 838 | 839 | 839 | 839 | 839 | 839 | 839 |
| 1.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 651 | 719 | 775 | 818 | 848 | 867 | 876 | 880 | 882 | 883 | 883 | 883 | 883 | 883 |
| 1.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 722 | 782 | 830 | 867 | 891 | 905 | 912 | 915 | 917 | 917 | 917 | 917 | 917 |
| 1.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 723 | 785 | 836 | 876 | 905 | 924 | 934 | 940 | 942 | 943 | 943 | 943 | 943 |
| 1.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 838 | 880 | 912 | 934 | 948 | 956 | 960 | 961 | 962 | 962 | 962 |
| 2.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 882 | 915 | 940 | 956 | 966 | 971 | 974 | 975 | 975 | 975 |
| 2.2 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 883 | 917 | 942 | 960 | 971 | 978 | 981 | 983 | 984 | 984 |
| 2.4 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 883 | 917 | 943 | 961 | 974 | 981 | 986 | 988 | 989 | 989 |
| 2.6 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 883 | 917 | 943 | 962 | 975 | 983 | 988 | 991 | 992 | 993 |
| 2.8 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 883 | 917 | 943 | 962 | 975 | 984 | 989 | 992 | 994 | 995 |
| 3.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 883 | 917 | 943 | 962 | 975 | 984 | 989 | 993 | 995 | 996 |

First, the standardized acceptance limits must be calculated from the following:

$$Fu = \frac{16 - (XBARF)}{(STDF)} = \frac{16 - 10.55}{0.84} = +6.5$$

$$Fl = \frac{8 - (XBARF)}{(STDF)} = \frac{8 - 10.55}{0.84} = -3.0$$

$$Vu = \frac{5 - (XBARV)}{(STDV)} = \frac{5 - 3.47}{0.62} = +2.5$$

$$Vl = \frac{2 - (XBARV)}{(STDV)} = \frac{2 - 3.47}{0.62} = -2.4$$

Once the standardized acceptance limits are determined, the 4 areas defined above can be calculated from the bivariate normal table for a correlation coefficient, P, of -0.4. The areas determined for the example problem are:

(Fu,Vu) = 98.95 [by interpolating between Z2=2.4 and 2.6 on the Z1=3.0 row (3.0 is the largest value used in the tables)]

(Fl,Vl) = 0.00 [from the cell with Z1=-3.0 and Z2=-2.4]

(Fl,Vu) = 0.00

(Fu,Vl) = 0.70

The PWL value is, therefore,

PWL = (Fu,Vu)+(Fl,Vl)-(Fl,Vu)-(Fu,Vl)

PWL = 98.95 + 0.00 - 0.00 - 0.70

PWL = 98.15

# APPENDIX B

## BIVARIATE NORMAL TABLES

This appendix presents tables for determining the total volume of the bivariate distribution that falls between 2 sets of upper and lower limits. For the case considered in this research, the upper and lower limits are those for Marshall flow and air voids. The PWL value for the population can be determined by calculating the two means and standard deviations and the one corelation coefficient for flow and air voids, and then using the tables in this appendix. An example illustrating the use of the bivariate tables for determining PWL is presented below.

As developed in Chapter V, 4 separate values must be determined from the bivariate tables to calculate each PWL value. The steps in the procedure for determining the bivariate PWL are as follows:

1. determine the mean and standard deviation values for flow and air voids, and the correlation coefficient between flow and air voids,

2. determine the upper and lower standardized limits for flow and for air voids,

3. designate the upper and lower standardized limits for flow as Fu and Fl, and designate the upper and lower standardized limits for air voids as Vu and Vl,

4. determine the 4 volumes (Fu,Vu) (Fl,Vl) (Fu,Vl) and (Fl,Vu) using the bivariate table for the value of the correlation coefficient, P, and

5. calculate PWL from PWL = (Fu,Vu)+(Fl,Vl)-(Fl,Vu)-(Fu,Vl).

A numerical example will help to illustrate the proper use of the tables. Let,

XBARF = 10.55 be the mean for flow,

STDF = 0.84 be the standard deviation for flow,

XBARV = 3.47 be the mean for air voids,

STDV = 0.62 be the standard deviation for air voids and

P = -0.40 be the correlation between flow and air voids,

then the bivariate PWL value can be calculated as follows.

APPENDIX A

AREAS UNDER THE STANDARD NORMAL DISTRIBUTION FROM $-\infty$ to Z.(23)

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

BIVARIATE NORMAL TABLES
ρ=-0.2

| z1 \ z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 10 | 12 | 14 | 15 | 17 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 4 | 5 | 7 | 9 | 11 | 14 | 17 | 20 | 22 | 25 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 8 | 11 | 14 | 18 | 22 | 27 | 31 | 35 | 39 | 42 | 45 | 48 | 49 | 51 | 52 | 52 | 53 | 53 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 7 | 9 | 13 | 17 | 22 | 28 | 34 | 41 | 47 | 53 | 59 | 64 | 68 | 71 | 74 | 76 | 77 | 78 | 78 | 79 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 7 | 10 | 14 | 19 | 26 | 33 | 42 | 51 | 60 | 69 | 78 | 85 | 92 | 98 | 102 | 106 | 109 | 110 | 112 | 112 | 113 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 10 | 15 | 21 | 28 | 37 | 48 | 60 | 72 | 85 | 97 | 109 | 120 | 129 | 136 | 143 | 147 | 151 | 153 | 155 | 156 | 156 | 157 |
| -0.8 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 14 | 21 | 29 | 39 | 52 | 66 | 82 | 99 | 116 | 133 | 148 | 162 | 174 | 184 | 192 | 198 | 202 | 205 | 207 | 209 | 209 | 210 |
| -0.6 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 8 | 13 | 19 | 28 | 39 | 53 | 70 | 89 | 110 | 132 | 154 | 175 | 195 | 213 | 228 | 240 | 250 | 257 | 263 | 266 | 269 | 270 | 271 | 272 |
| -0.4 | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 11 | 17 | 26 | 37 | 52 | 70 | 92 | 116 | 142 | 170 | 197 | 224 | 249 | 270 | 289 | 304 | 316 | 325 | 331 | 336 | 338 | 340 | 342 | 342 |
| -0.2 | 0 | 0 | 1 | 2 | 3 | 5 | 9 | 14 | 22 | 33 | 48 | 66 | 89 | 116 | 146 | 178 | 212 | 246 | 278 | 308 | 334 | 356 | 373 | 387 | 398 | 405 | 411 | 414 | 416 | 417 | 418 |
| 0.0 | 0 | 0 | 1 | 2 | 4 | 7 | 11 | 18 | 28 | 42 | 60 | 82 | 110 | 142 | 178 | 217 | 257 | 297 | 335 | 370 | 400 | 426 | 446 | 462 | 474 | 483 | 489 | 493 | 495 | 496 | 497 |
| 0.2 | 0 | 0 | 1 | 3 | 5 | 8 | 14 | 22 | 34 | 51 | 72 | 99 | 132 | 170 | 212 | 257 | 304 | 350 | 393 | 433 | 468 | 496 | 520 | 538 | 551 | 560 | 567 | 571 | 574 | 575 | 576 |
| 0.4 | 0 | 1 | 1 | 3 | 6 | 10 | 17 | 27 | 41 | 60 | 85 | 116 | 154 | 197 | 246 | 297 | 350 | 402 | 450 | 495 | 533 | 565 | 591 | 610 | 625 | 635 | 642 | 647 | 650 | 651 | 652 |
| 0.6 | 0 | 1 | 2 | 4 | 7 | 12 | 20 | 31 | 47 | 69 | 97 | 133 | 175 | 224 | 278 | 335 | 393 | 450 | 504 | 552 | 594 | 629 | 656 | 678 | 693 | 704 | 712 | 717 | 720 | 721 | 723 |
| 0.8 | 0 | 1 | 2 | 4 | 8 | 14 | 22 | 35 | 53 | 78 | 109 | 148 | 195 | 249 | 308 | 370 | 433 | 495 | 552 | 604 | 649 | 686 | 715 | 738 | 754 | 766 | 773 | 779 | 782 | 784 | 785 |
| 1.0 | 0 | 1 | 2 | 5 | 9 | 15 | 25 | 39 | 59 | 85 | 120 | 162 | 213 | 270 | 334 | 400 | 468 | 511 | 594 | 649 | 696 | 734 | 765 | 789 | 806 | 818 | 826 | 831 | 835 | 837 | 838 |
| 1.2 | 0 | 1 | 2 | 5 | 10 | 17 | 27 | 42 | 64 | 92 | 129 | 174 | 228 | 289 | 356 | 426 | 496 | 565 | 629 | 686 | 734 | 775 | 807 | 831 | 849 | 861 | 869 | 875 | 878 | 880 | 881 |
| 1.4 | 0 | 1 | 3 | 6 | 10 | 18 | 29 | 45 | 68 | 98 | 136 | 184 | 240 | 304 | 373 | 446 | 520 | 591 | 656 | 715 | 765 | 807 | 839 | 864 | 882 | 895 | 903 | 909 | 912 | 914 | 916 |
| 1.6 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 48 | 71 | 102 | 143 | 192 | 250 | 336 | 387 | 462 | 538 | 610 | 678 | 738 | 789 | 831 | 864 | 889 | 908 | 920 | 929 | 935 | 938 | 940 | 941 |
| 1.8 | 0 | 1 | 3 | 6 | 11 | 20 | 32 | 49 | 74 | 106 | 147 | 198 | 257 | 325 | 398 | 474 | 551 | 625 | 693 | 754 | 806 | 849 | 882 | 908 | 926 | 939 | 948 | 953 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 51 | 76 | 109 | 151 | 202 | 263 | 331 | 405 | 483 | 560 | 635 | 704 | 766 | 818 | 861 | 895 | 920 | 939 | 952 | 961 | 967 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 52 | 77 | 110 | 153 | 205 | 266 | 336 | 411 | 489 | 567 | 642 | 712 | 773 | 826 | 869 | 903 | 929 | 948 | 961 | 970 | 975 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 155 | 207 | 269 | 338 | 414 | 493 | 571 | 647 | 717 | 779 | 831 | 875 | 909 | 935 | 953 | 967 | 975 | 981 | 984 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 156 | 09 | 270 | 340 | 416 | 495 | 574 | 650 | 720 | 782 | 835 | 878 | 912 | 938 | 957 | 970 | 979 | 984 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 342 | 417 | 496 | 575 | 651 | 721 | 784 | 837 | 880 | 914 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 723 | 785 | 838 | 881 | 916 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
$\rho = -0.3$

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 13 | 15 | 16 | 18 | 19 | 20 | 20 | 21 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 5 | 7 | 9 | 11 | 14 | 17 | 19 | 22 | 25 | 27 | 29 | 31 | 32 | 33 | 33 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 8 | 11 | 14 | 18 | 22 | 27 | 31 | 35 | 39 | 43 | 46 | 48 | 50 | 51 | 52 | 52 | 53 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 9 | 13 | 17 | 23 | 28 | 35 | 41 | 48 | 54 | 60 | 65 | 69 | 72 | 74 | 76 | 77 | 78 | 79 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 7 | 10 | 14 | 20 | 26 | 34 | 43 | 52 | 62 | 71 | 79 | 87 | 94 | 99 | 104 | 107 | 109 | 111 | 112 | 113 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 4 | 7 | 10 | 15 | 22 | 29 | 39 | 50 | 62 | 75 | 88 | 101 | 112 | 123 | 132 | 139 | 145 | 149 | 152 | 154 | 155 | 156 | 157 |
| -0.8 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 10 | 15 | 22 | 31 | 42 | 56 | 71 | 87 | 105 | 122 | 139 | 154 | 167 | 179 | 188 | 195 | 200 | 204 | 206 | 208 | 209 | 210 |
| -0.6 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 6 | 9 | 14 | 22 | 31 | 43 | 59 | 76 | 97 | 118 | 141 | 163 | 184 | 204 | 220 | 234 | 246 | 254 | 261 | 265 | 268 | 270 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 1 | 1 | 3 | 5 | 8 | 13 | 20 | 29 | 42 | 59 | 78 | 102 | 127 | 155 | 183 | 211 | 237 | 261 | 281 | 298 | 311 | 322 | 329 | 334 | 338 | 340 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 1 | 2 | 4 | 7 | 11 | 17 | 26 | 39 | 56 | 76 | 102 | 130 | 163 | 196 | 231 | 265 | 296 | 324 | 348 | 368 | 383 | 395 | 403 | 409 | 413 | 416 | 417 | 418 |
| 0.0 | 0 | 0 | 1 | 1 | 3 | 5 | 9 | 14 | 23 | 34 | 50 | 71 | 97 | 127 | 163 | 201 | 242 | 282 | 322 | 358 | 391 | 418 | 441 | 456 | 472 | 481 | 488 | 492 | 495 | 496 | 497 |
| 0.2 | 0 | 0 | 1 | 2 | 4 | 6 | 11 | 18 | 28 | 43 | 62 | 87 | 118 | 155 | 196 | 242 | 289 | 336 | 381 | 422 | 459 | 490 | 515 | 534 | 549 | 559 | 566 | 571 | 574 | 575 | 576 |
| 0.4 | 0 | 0 | 1 | 2 | 5 | 8 | 14 | 22 | 35 | 52 | 75 | 105 | 141 | 183 | 231 | 282 | 336 | 389 | 439 | 485 | 525 | 559 | 586 | 607 | 623 | 634 | 641 | 646 | 650 | 651 | 652 |
| 0.6 | 0 | 0 | 1 | 3 | 6 | 10 | 17 | 27 | 41 | 62 | 88 | 122 | 163 | 211 | 265 | 322 | 381 | 439 | 494 | 544 | 588 | 624 | 653 | 675 | 692 | 703 | 711 | 716 | 720 | 722 | 723 |
| 0.8 | 0 | 1 | 2 | 3 | 7 | 12 | 19 | 31 | 48 | 71 | 101 | 139 | 184 | 237 | 296 | 358 | 422 | 485 | 544 | 597 | 643 | 682 | 712 | 736 | 753 | 765 | 773 | 778 | 782 | 784 | 785 |
| 1.0 | 0 | 1 | 2 | 4 | 8 | 13 | 22 | 35 | 54 | 79 | 112 | 154 | 204 | 261 | 324 | 391 | 459 | 525 | 588 | 643 | 692 | 732 | 763 | 787 | 805 | 818 | 826 | 831 | 835 | 837 | 838 |
| 1.2 | 0 | 1 | 2 | 5 | 9 | 15 | 25 | 39 | 60 | 87 | 123 | 167 | 220 | 281 | 348 | 418 | 490 | 559 | 624 | 682 | 732 | 773 | 805 | 830 | 848 | 861 | 869 | 875 | 878 | 880 | 881 |
| 1.4 | 0 | 1 | 2 | 5 | 9 | 16 | 27 | 43 | 65 | 94 | 132 | 179 | 234 | 298 | 368 | 441 | 515 | 586 | 653 | 712 | 763 | 805 | 838 | 864 | 882 | 895 | 903 | 909 | 912 | 914 | 916 |
| 1.6 | 0 | 1 | 3 | 5 | 10 | 18 | 29 | 46 | 69 | 99 | 139 | 188 | 246 | 311 | 383 | 458 | 534 | 607 | 675 | 736 | 787 | 830 | 864 | 889 | 908 | 920 | 929 | 935 | 938 | 940 | 941 |
| 1.8 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 48 | 72 | 104 | 145 | 195 | 254 | 322 | 395 | 472 | 549 | 623 | 692 | 753 | 805 | 848 | 882 | 908 | 926 | 939 | 948 | 954 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 6 | 11 | 20 | 32 | 50 | 74 | 107 | 149 | 200 | 261 | 329 | 403 | 481 | 559 | 634 | 703 | 765 | 818 | 861 | 895 | 920 | 939 | 952 | 961 | 967 | 970 | 972 | 973 |
| 2.2 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 51 | 76 | 109 | 152 | 204 | 265 | 334 | 409 | 488 | 566 | 641 | 711 | 773 | 826 | 869 | 903 | 929 | 948 | 961 | 970 | 975 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 7 | 12 | 21 | 33 | 52 | 77 | 111 | 154 | 206 | 268 | 338 | 413 | 492 | 571 | 646 | 716 | 778 | 831 | 875 | 909 | 935 | 954 | 967 | 975 | 981 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 155 | 208 | 270 | 340 | 416 | 495 | 574 | 650 | 720 | 782 | 835 | 878 | 912 | 938 | 957 | 970 | 979 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 271 | 341 | 417 | 496 | 575 | 651 | 722 | 784 | 837 | 880 | 914 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 418 | 497 | 576 | 652 | 723 | 785 | 838 | 881 | 916 | 941 | 960 | 973 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES

P=-0.4

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 15 | 16 | 18 | 19 | 20 | 20 | 21 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 8 | 11 | 13 | 16 | 19 | 22 | 25 | 27 | 29 | 31 | 32 | 33 | 34 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 5 | 8 | 10 | 14 | 18 | 22 | 27 | 31 | 36 | 40 | 43 | 46 | 48 | 50 | 51 | 52 | 53 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 13 | 17 | 22 | 29 | 35 | 42 | 49 | 55 | 61 | 66 | 70 | 73 | 75 | 77 | 78 | 78 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 4 | 6 | 10 | 14 | 20 | 27 | 35 | 44 | 54 | 63 | 73 | 82 | 89 | 96 | 101 | 105 | 108 | 110 | 112 | 112 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 10 | 15 | 22 | 31 | 41 | 52 | 65 | 78 | 92 | 105 | 116 | 126 | 135 | 142 | 147 | 150 | 153 | 155 | 156 | 156 |
| -0.8 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 10 | 16 | 23 | 33 | 45 | 59 | 75 | 93 | 111 | 128 | 145 | 160 | 173 | 183 | 192 | 198 | 202 | 206 | 208 | 209 | 210 |
| -0.6 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 10 | 15 | 23 | 34 | 47 | 64 | 83 | 104 | 127 | 150 | 173 | 194 | 212 | 228 | 241 | 251 | 258 | 264 | 267 | 270 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 9 | 14 | 22 | 33 | 47 | 65 | 87 | 112 | 139 | 168 | 197 | 225 | 251 | 273 | 292 | 307 | 319 | 327 | 333 | 337 | 340 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 8 | 13 | 20 | 31 | 45 | 64 | 87 | 115 | 146 | 180 | 216 | 251 | 284 | 314 | 340 | 362 | 379 | 392 | 402 | 408 | 413 | 415 | 417 | 418 |
| 0.0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 17 | 27 | 41 | 59 | 83 | 112 | 146 | 184 | 225 | 267 | 308 | 347 | 382 | 411 | 436 | 455 | 469 | 480 | 487 | 491 | 494 | 496 | 497 |
| 0.2 | 0 | 0 | 1 | 1 | 2 | 5 | 8 | 14 | 22 | 35 | 52 | 75 | 104 | 139 | 180 | 225 | 273 | 321 | 368 | 412 | 451 | 483 | 510 | 531 | 547 | 558 | 565 | 570 | 574 | 575 | 577 |
| 0.4 | 0 | 0 | 1 | 2 | 3 | 6 | 11 | 18 | 29 | 44 | 65 | 93 | 127 | 168 | 216 | 267 | 321 | 376 | 428 | 476 | 518 | 554 | 583 | 605 | 621 | 633 | 641 | 646 | 649 | 651 | 653 |
| 0.6 | 0 | 0 | 1 | 2 | 4 | 8 | 13 | 22 | 35 | 54 | 78 | 111 | 150 | 197 | 251 | 308 | 368 | 428 | 485 | 537 | 582 | 620 | 650 | 673 | 691 | 703 | 711 | 716 | 720 | 722 | 723 |
| 0.8 | 0 | 0 | 1 | 3 | 5 | 10 | 16 | 27 | 42 | 63 | 92 | 128 | 173 | 225 | 284 | 347 | 412 | 476 | 537 | 591 | 639 | 679 | 710 | 734 | 752 | 765 | 773 | 779 | 782 | 784 | 785 |
| 1.0 | 0 | 1 | 2 | 3 | 6 | 11 | 19 | 31 | 49 | 73 | 105 | 145 | 194 | 251 | 314 | 382 | 451 | 518 | 582 | 639 | 688 | 729 | 762 | 787 | 805 | 817 | 826 | 832 | 835 | 837 | 838 |
| 1.2 | 0 | 1 | 2 | 4 | 7 | 13 | 22 | 36 | 55 | 82 | 116 | 160 | 212 | 273 | 340 | 411 | 483 | 554 | 620 | 679 | 729 | 771 | 804 | 830 | 848 | 861 | 869 | 875 | 878 | 880 | 882 |
| 1.4 | 0 | 1 | 2 | 4 | 8 | 15 | 25 | 40 | 61 | 89 | 126 | 173 | 228 | 292 | 362 | 436 | 510 | 583 | 650 | 710 | 762 | 804 | 838 | 863 | 882 | 895 | 904 | 909 | 913 | 915 | 916 |
| 1.6 | 0 | 1 | 2 | 5 | 9 | 16 | 27 | 43 | 66 | 96 | 135 | 183 | 241 | 307 | 379 | 455 | 531 | 605 | 673 | 734 | 787 | 830 | 863 | 889 | 908 | 921 | 929 | 935 | 938 | 940 | 942 |
| 1.8 | 0 | 1 | 3 | 5 | 10 | 18 | 29 | 46 | 70 | 101 | 142 | 192 | 251 | 319 | 392 | 469 | 547 | 621 | 691 | 752 | 805 | 848 | 882 | 908 | 926 | 939 | 948 | 954 | 957 | 959 | 960 |
| 2.0 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 48 | 73 | 105 | 147 | 198 | 258 | 327 | 402 | 480 | 558 | 633 | 703 | 765 | 817 | 861 | 895 | 921 | 939 | 952 | 961 | 967 | 970 | 972 | 974 |
| 2.2 | 0 | 1 | 3 | 6 | 11 | 20 | 32 | 50 | 75 | 108 | 150 | 202 | 264 | 333 | 408 | 487 | 565 | 641 | 711 | 773 | 826 | 869 | 904 | 929 | 948 | 961 | 970 | 976 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 51 | 77 | 110 | 153 | 206 | 267 | 337 | 413 | 491 | 570 | 646 | 716 | 779 | 832 | 875 | 909 | 935 | 954 | 967 | 976 | 981 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 155 | 208 | 270 | 340 | 415 | 494 | 574 | 649 | 720 | 782 | 835 | 878 | 913 | 938 | 957 | 970 | 979 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 78 | 112 | 156 | 209 | 271 | 341 | 417 | 496 | 575 | 651 | 722 | 784 | 837 | 880 | 915 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 210 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 882 | 916 | 942 | 960 | 974 | 982 | 988 | 991 | 993 | 995 |

BIVARIATE NORMAL TABLES
P=-0.5

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 16 | 18 | 19 | 20 | 20 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 8 | 10 | 13 | 16 | 19 | 22 | 25 | 27 | 30 | 31 | 32 | 33 | 34 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 7 | 10 | 13 | 17 | 22 | 27 | 32 | 36 | 40 | 44 | 47 | 49 | 50 | 52 | 52 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 12 | 17 | 22 | 29 | 36 | 43 | 50 | 56 | 62 | 67 | 71 | 74 | 76 | 77 | 78 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 14 | 20 | 27 | 36 | 45 | 55 | 65 | 75 | 84 | 92 | 98 | 103 | 107 | 109 | 111 | 112 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 10 | 15 | 22 | 31 | 42 | 54 | 68 | 82 | 96 | 109 | 120 | 130 | 138 | 144 | 149 | 152 | 154 | 156 | 156 |
| -0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 10 | 16 | 24 | 34 | 47 | 63 | 80 | 98 | 117 | 135 | 152 | 166 | 179 | 188 | 196 | 201 | 205 | 207 | 209 | 209 |
| -0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 16 | 25 | 36 | 51 | 69 | 90 | 112 | 137 | 161 | 183 | 204 | 222 | 236 | 248 | 256 | 262 | 267 | 269 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 9 | 15 | 24 | 36 | 52 | 72 | 96 | 123 | 152 | 183 | 212 | 240 | 265 | 286 | 303 | 316 | 325 | 332 | 336 | 339 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 5 | 8 | 14 | 22 | 34 | 51 | 72 | 98 | 129 | 163 | 199 | 236 | 271 | 304 | 332 | 356 | 375 | 390 | 400 | 407 | 412 | 415 | 417 | 418 |
| 0.0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 7 | 12 | 20 | 31 | 47 | 69 | 96 | 129 | 167 | 208 | 251 | 294 | 335 | 372 | 404 | 431 | 451 | 467 | 478 | 486 | 491 | 494 | 496 | 497 |
| 0.2 | 0 | 0 | 0 | 1 | 1 | 3 | 5 | 10 | 17 | 27 | 42 | 63 | 90 | 123 | 163 | 208 | 257 | 307 | 356 | 402 | 443 | 478 | 506 | 528 | 545 | 557 | 565 | 570 | 573 | 575 | 577 |
| 0.4 | 0 | 0 | 0 | 1 | 2 | 4 | 8 | 13 | 22 | 36 | 54 | 80 | 112 | 152 | 199 | 251 | 307 | 363 | 417 | 467 | 512 | 549 | 579 | 603 | 620 | 632 | 641 | 646 | 650 | 652 | 653 |
| 0.6 | 0 | 0 | 1 | 1 | 3 | 6 | 10 | 17 | 29 | 45 | 68 | 98 | 137 | 183 | 236 | 294 | 356 | 417 | 476 | 530 | 577 | 616 | 648 | 672 | 690 | 702 | 711 | 716 | 720 | 722 | 723 |
| 0.8 | 0 | 0 | 1 | 2 | 4 | 7 | 13 | 22 | 36 | 55 | 82 | 117 | 161 | 212 | 271 | 335 | 402 | 467 | 530 | 586 | 635 | 676 | 709 | 734 | 752 | 765 | 773 | 779 | 782 | 784 | 785 |
| 1.0 | 0 | 0 | 1 | 2 | 5 | 9 | 16 | 27 | 43 | 65 | 96 | 135 | 183 | 240 | 304 | 372 | 443 | 512 | 577 | 635 | 686 | 728 | 761 | 786 | 805 | 818 | 826 | 832 | 835 | 837 | 838 |
| 1.2 | 0 | 0 | 1 | 3 | 6 | 11 | 19 | 32 | 50 | 75 | 109 | 152 | 204 | 265 | 332 | 404 | 478 | 549 | 616 | 676 | 728 | 770 | 804 | 829 | 848 | 861 | 870 | 875 | 879 | 881 | 882 |
| 1.4 | 0 | 1 | 2 | 4 | 7 | 13 | 22 | 36 | 56 | 84 | 120 | 166 | 222 | 286 | 356 | 431 | 506 | 579 | 648 | 709 | 761 | 804 | 830 | 863 | 882 | 895 | 904 | 909 | 913 | 915 | 916 |
| 1.6 | 0 | 1 | 2 | 4 | 8 | 15 | 25 | 40 | 62 | 92 | 130 | 179 | 236 | 303 | 375 | 451 | 528 | 603 | 672 | 734 | 786 | 829 | 863 | 889 | 908 | 921 | 930 | 935 | 939 | 941 | 942 |
| 1.8 | 0 | 1 | 2 | 5 | 9 | 16 | 27 | 44 | 67 | 98 | 138 | 188 | 248 | 316 | 390 | 467 | 545 | 620 | 690 | 752 | 805 | 848 | 882 | 908 | 927 | 940 | 948 | 954 | 957 | 959 | 961 |
| 2.0 | 0 | 1 | 3 | 5 | 10 | 18 | 30 | 47 | 71 | 103 | 144 | 196 | 256 | 325 | 400 | 478 | 557 | 632 | 702 | 765 | 818 | 861 | 895 | 921 | 940 | 953 | 961 | 967 | 970 | 973 | 974 |
| 2.2 | 0 | 1 | 3 | 6 | 11 | 19 | 31 | 49 | 74 | 107 | 149 | 201 | 262 | 332 | 407 | 486 | 565 | 641 | 711 | 773 | 826 | 870 | 904 | 930 | 948 | 961 | 970 | 976 | 979 | 981 | 982 |
| 2.4 | 0 | 1 | 3 | 6 | 11 | 20 | 32 | 50 | 76 | 109 | 152 | 205 | 267 | 336 | 412 | 491 | 570 | 646 | 716 | 779 | 832 | 875 | 909 | 935 | 954 | 967 | 976 | 981 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 52 | 77 | 111 | 154 | 207 | 269 | 339 | 415 | 494 | 573 | 650 | 720 | 782 | 835 | 879 | 913 | 939 | 957 | 970 | 979 | 985 | 988 | 990 | 991 |
| 2.8 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 52 | 78 | 112 | 156 | 209 | 271 | 341 | 417 | 496 | 575 | 652 | 722 | 784 | 837 | 881 | 915 | 941 | 959 | 973 | 981 | 987 | 990 | 992 | 994 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 838 | 882 | 916 | 942 | 961 | 974 | 982 | 988 | 991 | 994 | 995 |

BIVARIATE NORMAL TABLES
P=-0.6

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 7 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 4 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 18 | 19 | 20 | 21 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 7 | 9 | 12 | 16 | 19 | 22 | 25 | 28 | 30 | 32 | 33 | 33 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 13 | 17 | 22 | 27 | 32 | 37 | 41 | 45 | 48 | 50 | 51 | 52 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 11 | 16 | 22 | 29 | 36 | 44 | 51 | 58 | 64 | 68 | 72 | 75 | 77 | 78 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 8 | 13 | 19 | 27 | 36 | 46 | 57 | 68 | 78 | 87 | 95 | 101 | 105 | 108 | 111 | 112 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 9 | 15 | 22 | 31 | 43 | 56 | 71 | 86 | 100 | 114 | 125 | 135 | 142 | 148 | 151 | 154 | 155 | 156 |
| -0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 16 | 24 | 35 | 49 | 66 | 85 | 104 | 120 | 143 | 159 | 174 | 185 | 194 | 200 | 204 | 207 | 208 | 209 |
| -0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 16 | 25 | 38 | 54 | 74 | 97 | 121 | 147 | 172 | 195 | 215 | 232 | 245 | 254 | 261 | 266 | 269 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 5 | 9 | 16 | 25 | 39 | 57 | 79 | 106 | 135 | 167 | 199 | 229 | 256 | 279 | 298 | 313 | 324 | 331 | 336 | 339 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 5 | | 15 | 24 | 38 | 57 | 81 | 110 | 144 | 182 | 220 | 258 | 293 | 325 | 351 | 372 | 387 | 399 | 407 | 412 | 415 | 417 | 418 |
| 0.0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 7 | 13 | 22 | 35 | 54 | 79 | 110 | 148 | 190 | 234 | 280 | 323 | 363 | 398 | 426 | 449 | 465 | 477 | 486 | 491 | 494 | 496 | 497 |
| 0.2 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 11 | 19 | 31 | 49 | 74 | 106 | 144 | 190 | 239 | 291 | 343 | 391 | 435 | 472 | 503 | 526 | 544 | 556 | 565 | 570 | 574 | 576 | 577 |
| 0.4 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 9 | 16 | 27 | 43 | 66 | 97 | 135 | 182 | 234 | 291 | 349 | 406 | 459 | 506 | 545 | 577 | 602 | 620 | 632 | 641 | 646 | 650 | 652 | 653 |
| 0.6 | 0 | 0 | 0 | 1 | 2 | 4 | 7 | 13 | 22 | 36 | 56 | 85 | 121 | 167 | 220 | 280 | 343 | 406 | 467 | 523 | 572 | 613 | 646 | 671 | 690 | 702 | 711 | 717 | 720 | 722 | 723 |
| 0.8 | 0 | 0 | 0 | 1 | 3 | 5 | 9 | 17 | 29 | 46 | 71 | 104 | 147 | 199 | 258 | 323 | 391 | 459 | 523 | 582 | 632 | 674 | 708 | 733 | 752 | 765 | 773 | 779 | 782 | 784 | 785 |
| 1.0 | 0 | 0 | 1 | 2 | 3 | 7 | 12 | 22 | 36 | 57 | 86 | 124 | 172 | 229 | 293 | 363 | 435 | 506 | 572 | 632 | 684 | 727 | 761 | 786 | 805 | 818 | 826 | 832 | 835 | 837 | 839 |
| 1.2 | 0 | 0 | 1 | 2 | 5 | 9 | 16 | 27 | 44 | 68 | 100 | 143 | 195 | 256 | 325 | 398 | 472 | 545 | 613 | 674 | 727 | 770 | 804 | 830 | 848 | 861 | 870 | 876 | 879 | 881 | 882 |
| 1.4 | 0 | 0 | 1 | 3 | 6 | 11 | 19 | 32 | 51 | 78 | 114 | 159 | 215 | 279 | 351 | 426 | 503 | 577 | 646 | 708 | 761 | 804 | 838 | 864 | 882 | 895 | 904 | 910 | 913 | 915 | 916 |
| 1.6 | 0 | 1 | 2 | 4 | 7 | 13 | 22 | 37 | 58 | 87 | 125 | 174 | 232 | 298 | 372 | 449 | 526 | 602 | 671 | 733 | 786 | 830 | 864 | 890 | 908 | 921 | 930 | 936 | 939 | 941 | 942 |
| 1.8 | 0 | 1 | 2 | 4 | 8 | 15 | 25 | 41 | 64 | 95 | 135 | 185 | 245 | 313 | 387 | 465 | 544 | 620 | 690 | 752 | 805 | 848 | 882 | 908 | 927 | 940 | 949 | 954 | 958 | 960 | 961 |
| 2.0 | 0 | 1 | 2 | 5 | 9 | 17 | 28 | 45 | 68 | 101 | 142 | 194 | 254 | 324 | 399 | 477 | 556 | 632 | 702 | 765 | 818 | 861 | 895 | 921 | 940 | 953 | 962 | 967 | 971 | 973 | 974 |
| 2.2 | 0 | 1 | 2 | 5 | 10 | 18 | 30 | 48 | 72 | 105 | 148 | 200 | 261 | 331 | 407 | 486 | 565 | 641 | 711 | 773 | 826 | 870 | 904 | 930 | 949 | 962 | 970 | 976 | 980 | 982 | 983 |
| 2.4 | 0 | 1 | 3 | 6 | 11 | 19 | 32 | 50 | 75 | 108 | 151 | 204 | 266 | 336 | 412 | 491 | 570 | 646 | 717 | 779 | 832 | 876 | 910 | 936 | 954 | 967 | 976 | 982 | 985 | 987 | 988 |
| 2.6 | 0 | 1 | 3 | 6 | 11 | 20 | 33 | 51 | 77 | 111 | 154 | 207 | 269 | 339 | 415 | 494 | 574 | 650 | 720 | 782 | 835 | 879 | 913 | 939 | 958 | 971 | 980 | 985 | 989 | 991 | 992 |
| 2.8 | 0 | 1 | 3 | 6 | 12 | 21 | 33 | 52 | 78 | 112 | 155 | 208 | 271 | 341 | 417 | 496 | 576 | 652 | 722 | 784 | 837 | 881 | 915 | 941 | 960 | 973 | 982 | 987 | 991 | 993 | 994 |
| 3.0 | 0 | 1 | 3 | 7 | 12 | 21 | 34 | 53 | 79 | 113 | 156 | 209 | 272 | 342 | 418 | 497 | 577 | 653 | 723 | 785 | 839 | 882 | 916 | 942 | 961 | 974 | 983 | 988 | 992 | 994 | 995 |

BIVARIATE NORMAL TABLES
$\rho=-0.7$

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 6 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 | 11 | 12 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 13 | 15 | 17 | 18 | 19 | 20 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 6 | 9 | 12 | 15 | 19 | 23 | 26 | 29 | 31 | 32 | 33 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 8 | 11 | 16 | 21 | 27 | 33 | 38 | 42 | 46 | 49 | 51 | 52 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 15 | 21 | 28 | 37 | 45 | 53 | 60 | 66 | 70 | 74 | 76 | 78 | 78 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 7 | 12 | 18 | 26 | 36 | 47 | 59 | 71 | 81 | 91 | 98 | 104 | 108 | 110 | 112 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 8 | 13 | 21 | 31 | 44 | 59 | 75 | 91 | 106 | 120 | 131 | 140 | 146 | 151 | 153 | 155 | 156 |
| -0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 5 | 8 | 14 | 23 | 35 | 51 | 70 | 90 | 112 | 133 | 152 | 168 | 181 | 191 | 199 | 203 | 207 | 208 | 209 |
| -0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 9 | 15 | 25 | 39 | 57 | 79 | 105 | 132 | 159 | 185 | 208 | 227 | 242 | 253 | 261 | 266 | 269 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 8 | 15 | 25 | 40 | 61 | 86 | 116 | 149 | 184 | 217 | 247 | 273 | 294 | 311 | 322 | 330 | 336 | 339 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 8 | 14 | 25 | 40 | 62 | 90 | 124 | 162 | 203 | 244 | 283 | 317 | 346 | 369 | 386 | 398 | 406 | 412 | 415 | 417 | 418 |
| 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 7 | 13 | 23 | 39 | 61 | 90 | 127 | 169 | 216 | 264 | 311 | 354 | 392 | 423 | 447 | 464 | 477 | 486 | 491 | 494 | 496 | 498 |
| 0.2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 6 | 12 | 21 | 35 | 57 | 86 | 124 | 169 | 221 | 275 | 330 | 382 | 428 | 468 | 500 | 525 | 543 | 556 | 565 | 570 | 574 | 576 | 577 |
| 0.4 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 10 | 18 | 31 | 51 | 79 | 116 | 162 | 216 | 275 | 336 | 396 | 452 | 501 | 542 | 575 | 601 | 619 | 632 | 641 | 646 | 650 | 652 | 653 |
| 0.6 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 8 | 15 | 26 | 44 | 70 | 105 | 149 | 203 | 264 | 330 | 396 | 460 | 518 | 569 | 612 | 645 | 671 | 690 | 703 | 711 | 717 | 720 | 722 | 723 |
| 0.8 | 0 | 0 | 0 | 1 | 1 | 3 | 6 | 11 | 21 | 36 | 59 | 90 | 132 | 184 | 244 | 311 | 382 | 452 | 518 | 579 | 631 | 673 | 707 | 733 | 752 | 765 | 774 | 779 | 783 | 785 | 786 |
| 1.0 | 0 | 0 | 0 | 1 | 2 | 4 | 9 | 16 | 28 | 47 | 75 | 112 | 159 | 217 | 283 | 354 | 428 | 501 | 569 | 631 | 683 | 726 | 760 | 786 | 805 | 818 | 827 | 832 | 836 | 838 | 839 |
| 1.2 | 0 | 0 | 1 | 1 | 3 | 6 | 12 | 21 | 37 | 59 | 91 | 133 | 185 | 247 | 317 | 392 | 468 | 542 | 612 | 673 | 726 | 770 | 804 | 830 | 849 | 862 | 870 | 876 | 879 | 881 | 882 |
| 1.4 | 0 | 0 | 1 | 2 | 4 | 8 | 15 | 27 | 45 | 71 | 106 | 152 | 208 | 273 | 346 | 423 | 500 | 575 | 645 | 707 | 760 | 804 | 838 | 864 | 883 | 896 | 905 | 910 | 914 | 915 | 917 |
| 1.6 | 0 | 0 | 1 | 3 | 5 | 10 | 19 | 33 | 53 | 81 | 120 | 168 | 227 | 294 | 369 | 447 | 525 | 601 | 671 | 733 | 786 | 830 | 864 | 890 | 909 | 922 | 930 | 936 | 939 | 941 | 942 |
| 1.8 | 0 | 0 | 1 | 3 | 7 | 13 | 23 | 38 | 60 | 91 | 131 | 181 | 242 | 311 | 386 | 464 | 543 | 619 | 690 | 752 | 805 | 849 | 883 | 909 | 927 | 940 | 949 | 955 | 958 | 960 | 961 |
| 2.0 | 0 | 1 | 2 | 4 | 8 | 15 | 26 | 42 | 66 | 98 | 140 | 191 | 253 | 322 | 398 | 477 | 556 | 632 | 703 | 765 | 818 | 862 | 896 | 922 | 940 | 954 | 962 | 968 | 971 | 973 | 974 |
| 2.2 | 0 | 1 | 2 | 5 | 9 | 17 | 29 | 46 | 70 | 104 | 146 | 199 | 261 | 330 | 406 | 486 | 565 | 641 | 711 | 774 | 827 | 870 | 905 | 930 | 949 | 962 | 971 | 976 | 980 | 982 | 983 |
| 2.4 | 0 | 1 | 2 | 5 | 10 | 18 | 31 | 49 | 74 | 108 | 151 | 203 | 266 | 336 | 412 | 491 | 570 | 646 | 717 | 779 | 832 | 876 | 910 | 936 | 955 | 968 | 976 | 982 | 985 | 987 | 989 |
| 2.6 | 0 | 1 | 3 | 6 | 11 | 19 | 32 | 51 | 76 | 110 | 153 | 207 | 269 | 339 | 415 | 494 | 574 | 650 | 720 | 783 | 836 | 879 | 914 | 939 | 958 | 971 | 980 | 985 | 989 | 991 | 992 |
| 2.8 | 0 | 1 | 3 | 6 | 12 | 20 | 33 | 52 | 78 | 112 | 155 | 208 | 271 | 341 | 417 | 496 | 576 | 652 | 722 | 785 | 838 | 881 | 915 | 941 | 960 | 973 | 982 | 987 | 991 | 993 | 994 |
| 3.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 209 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 882 | 917 | 942 | 961 | 974 | 983 | 989 | 992 | 994 | 995 |

BIVARIATE NORMAL TABLES
P -0.9

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 5 | 6 | 6 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 6 | 8 | 10 | 11 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 9 | 13 | 16 | 18 | 20 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 9 | 14 | 19 | 24 | 28 | 31 | 33 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 7 | 12 | 20 | 28 | 36 | 42 | 47 | 50 | 52 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 9 | 16 | 26 | 38 | 50 | 60 | 67 | 73 | 76 | 78 | 79 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 11 | 21 | 34 | 51 | 67 | 82 | 93 | 101 | 107 | 110 | 112 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 6 | 13 | 26 | 43 | 64 | 87 | 107 | 124 | 136 | 145 | 150 | 154 | 156 | 157 |
| -0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 7 | 15 | 30 | 52 | 79 | 108 | 135 | 158 | 176 | 189 | 198 | 203 | 207 | 209 | 210 |
| -0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 7 | 17 | 34 | 60 | 93 | 129 | 164 | 195 | 220 | 238 | 251 | 260 | 266 | 269 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 8 | 18 | 37 | 66 | 104 | 148 | 192 | 231 | 264 | 290 | 309 | 322 | 331 | 336 | 340 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 8 | 18 | 39 | 70 | 113 | 163 | 216 | 264 | 306 | 340 | 366 | 385 | 398 | 407 | 412 | 416 | 418 | 419 |
| 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 7 | 18 | 39 | 72 | 118 | 173 | 233 | 291 | 342 | 385 | 419 | 445 | 464 | 477 | 486 | 492 | 495 | 497 | 498 |
| 0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 7 | 17 | 37 | 70 | 118 | 177 | 242 | 308 | 368 | 421 | 464 | 499 | 524 | 543 | 556 | 565 | 571 | 574 | 576 | 577 |
| 0.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 6 | 15 | 34 | 66 | 113 | 173 | 242 | 314 | 382 | 444 | 497 | 540 | 575 | 601 | 619 | 633 | 641 | 647 | 650 | 652 | 653 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 5 | 13 | 30 | 60 | 104 | 163 | 233 | 308 | 382 | 452 | 514 | 567 | 611 | 645 | 671 | 690 | 703 | 712 | 717 | 721 | 723 | 724 |
| 0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 4 | 11 | 26 | 52 | 93 | 148 | 216 | 291 | 368 | 444 | 514 | 576 | 630 | 673 | 707 | 733 | 752 | 765 | 774 | 780 | 783 | 785 | 786 |
| 1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 9 | 21 | 43 | 79 | 129 | 192 | 264 | 342 | 421 | 497 | 567 | 630 | 683 | 726 | 761 | 787 | 805 | 819 | 827 | 833 | 836 | 838 | 839 |
| 1.2 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 7 | 16 | 34 | 64 | 108 | 164 | 231 | 306 | 385 | 464 | 540 | 611 | 673 | 726 | 770 | 804 | 830 | 849 | 862 | 871 | 877 | 880 | 882 | 883 |
| 1.4 | 0 | 0 | 0 | 0 | 0 | 2 | 5 | 12 | 26 | 51 | 87 | 135 | 195 | 264 | 340 | 419 | 499 | 575 | 645 | 707 | 761 | 804 | 818 | 864 | 883 | 896 | 905 | 911 | 914 | 916 | 917 |
| 1.6 | 0 | 0 | 0 | 0 | 1 | 3 | 9 | 20 | 38 | 67 | 107 | 158 | 220 | 290 | 366 | 445 | 524 | 601 | 671 | 733 | 787 | 830 | 864 | 890 | 909 | 922 | 931 | 937 | 940 | 942 | 943 |
| 1.8 | 0 | 0 | 0 | 1 | 2 | 6 | 14 | 28 | 50 | 82 | 124 | 176 | 238 | 309 | 385 | 464 | 543 | 619 | 690 | 752 | 805 | 849 | 883 | 909 | 928 | 941 | 950 | 956 | 959 | 961 | 962 |
| 2.0 | 0 | 0 | 0 | 1 | 4 | 9 | 19 | 36 | 60 | 93 | 136 | 189 | 251 | 322 | 398 | 477 | 556 | 633 | 703 | 765 | 819 | 862 | 896 | 922 | 941 | 954 | 963 | 969 | 972 | 974 | 975 |
| 2.2 | 0 | 0 | 1 | 2 | 6 | 13 | 24 | 42 | 67 | 101 | 145 | 198 | 260 | 331 | 407 | 486 | 565 | 641 | 712 | 774 | 827 | 871 | 905 | 931 | 950 | 963 | 972 | 978 | 981 | 983 | 984 |
| 2.4 | 0 | 0 | 1 | 4 | 8 | 16 | 28 | 47 | 73 | 107 | 150 | 203 | 266 | 336 | 412 | 492 | 571 | 647 | 717 | 780 | 833 | 877 | 911 | 937 | 956 | 969 | 978 | 983 | 987 | 988 | 989 |
| 2.6 | 0 | 1 | 2 | 5 | 10 | 18 | 31 | 50 | 76 | 110 | 154 | 207 | 269 | 340 | 416 | 495 | 574 | 650 | 721 | 781 | 836 | 880 | 914 | 940 | 959 | 972 | 981 | 987 | 990 | 992 | 993 |
| 2.8 | 0 | 1 | 3 | 6 | 11 | 20 | 33 | 52 | 78 | 112 | 156 | 209 | 271 | 341 | 418 | 497 | 576 | 652 | 723 | 785 | 838 | 882 | 916 | 942 | 961 | 974 | 983 | 988 | 992 | 994 | 995 |
| 3.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 79 | 113 | 157 | 210 | 272 | 342 | 419 | 498 | 577 | 653 | 724 | 786 | 839 | 883 | 917 | 943 | 962 | 975 | 984 | 989 | 993 | 995 | 996 |

BIVARIATE NORMAL TABLES
$\rho = -0.8$

| Z1 \ Z2 | -3.0 | | | | | -2.0 | | | | | -1.0 | | | | | 0.0 | | | | | 1.0 | | | | | 2.0 | | | | | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -3.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -2.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| -2.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| -2.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 6 |
| -2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 11 | 12 |
| -2.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 5 | 7 | 10 | 13 | 15 | 17 | 19 | 20 | 21 |
| -1.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 4 | 7 | 11 | 15 | 19 | 23 | 27 | 30 | 32 | 33 | 34 |
| -1.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 15 | 21 | 27 | 34 | 39 | 44 | 48 | 50 | 52 | 53 |
| -1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 7 | 13 | 19 | 28 | 37 | 47 | 55 | 63 | 69 | 73 | 76 | 77 | 78 |
| -1.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 9 | 16 | 24 | 36 | 49 | 62 | 75 | 86 | 95 | 102 | 107 | 110 | 112 | 113 |
| -1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 6 | 11 | 18 | 30 | 44 | 61 | 79 | 97 | 114 | 127 | 138 | 145 | 150 | 153 | 155 | 156 |
| -0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 52 | 74 | 97 | 121 | 144 | 163 | 178 | 190 | 198 | 203 | 207 | 208 | 210 |
| -0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 6 | 13 | 23 | 38 | 59 | 85 | 114 | 145 | 175 | 201 | 223 | 240 | 252 | 260 | 266 | 269 | 271 | 272 |
| -0.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 6 | 13 | 24 | 41 | 64 | 94 | 129 | 167 | 204 | 239 | 268 | 291 | 309 | 322 | 330 | 336 | 339 | 341 | 342 |
| -0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 6 | 13 | 24 | 42 | 67 | 100 | 140 | 185 | 230 | 273 | 311 | 342 | 367 | 385 | 398 | 407 | 412 | 415 | 417 | 418 |
| 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 6 | 12 | 23 | 41 | 67 | 102 | 146 | 196 | 249 | 300 | 347 | 387 | 420 | 445 | 464 | 477 | 486 | 491 | 495 | 497 | 498 |
| 0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 5 | 11 | 21 | 38 | 64 | 100 | 146 | 200 | 258 | 318 | 373 | 423 | 465 | 499 | 525 | 543 | 556 | 565 | 571 | 574 | 576 | 577 |
| 0.4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 9 | 18 | 34 | 59 | 94 | 140 | 196 | 258 | 324 | 388 | 446 | 498 | 541 | 575 | 601 | 619 | 632 | 641 | 647 | 650 | 652 | 653 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 7 | 16 | 30 | 52 | 85 | 129 | 185 | 249 | 318 | 388 | 454 | 515 | 568 | 611 | 645 | 671 | 690 | 703 | 711 | 717 | 720 | 722 | 723 |
| 0.8 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 6 | 13 | 24 | 44 | 74 | 114 | 167 | 230 | 300 | 373 | 446 | 515 | 577 | 630 | 673 | 707 | 733 | 752 | 765 | 774 | 779 | 783 | 785 | 786 |
| 1.0 | 0 | 0 | 0 | 0 | 1 | 2 | 4 | 10 | 19 | 36 | 61 | 97 | 145 | 204 | 273 | 347 | 423 | 498 | 568 | 630 | 683 | 726 | 761 | 786 | 805 | 818 | 827 | 833 | 836 | 838 | 839 |
| 1.2 | 0 | 0 | 0 | 0 | 1 | 3 | 7 | 15 | 28 | 49 | 79 | 121 | 175 | 239 | 311 | 387 | 465 | 541 | 611 | 673 | 726 | 770 | 804 | 830 | 849 | 862 | 871 | 876 | 880 | 882 | 883 |
| 1.4 | 0 | 0 | 0 | 1 | 2 | 5 | 11 | 21 | 37 | 62 | 97 | 144 | 201 | 268 | 342 | 420 | 499 | 575 | 645 | 707 | 761 | 804 | 838 | 864 | 883 | 896 | 905 | 910 | 914 | 916 | 917 |
| 1.6 | 0 | 0 | 1 | 1 | 3 | 7 | 15 | 27 | 47 | 75 | 114 | 163 | 223 | 291 | 367 | 445 | 525 | 601 | 671 | 733 | 786 | 830 | 864 | 890 | 909 | 922 | 931 | 936 | 940 | 942 | 943 |
| 1.8 | 0 | 0 | 1 | 2 | 5 | 10 | 19 | 34 | 55 | 86 | 127 | 178 | 240 | 309 | 385 | 464 | 543 | 619 | 690 | 752 | 805 | 849 | 883 | 909 | 928 | 941 | 950 | 955 | 959 | 961 | 962 |
| 2.0 | 0 | 0 | 1 | 3 | 6 | 13 | 23 | 39 | 63 | 95 | 138 | 190 | 252 | 322 | 398 | 477 | 556 | 632 | 703 | 765 | 818 | 862 | 896 | 922 | 941 | 954 | 963 | 968 | 972 | 974 | 975 |
| 2.2 | 0 | 1 | 2 | 4 | 8 | 15 | 27 | 44 | 69 | 102 | 145 | 198 | 260 | 330 | 407 | 486 | 565 | 641 | 711 | 774 | 827 | 871 | 905 | 931 | 950 | 963 | 971 | 977 | 980 | 982 | 983 |
| 2.4 | 0 | 1 | 2 | 5 | 9 | 17 | 30 | 48 | 73 | 107 | 150 | 203 | 266 | 336 | 412 | 491 | 571 | 647 | 717 | 779 | 833 | 876 | 910 | 936 | 955 | 968 | 977 | 983 | 986 | 988 | 989 |
| 2.6 | 0 | 1 | 2 | 5 | 11 | 19 | 32 | 50 | 76 | 110 | 153 | 207 | 269 | 339 | 415 | 495 | 574 | 650 | 720 | 783 | 836 | 880 | 914 | 940 | 959 | 972 | 980 | 986 | 989 | 991 | 992 |
| 2.8 | 0 | 1 | 3 | 6 | 11 | 20 | 33 | 52 | 77 | 112 | 155 | 208 | 271 | 341 | 417 | 497 | 576 | 652 | 722 | 785 | 838 | 882 | 916 | 942 | 961 | 974 | 982 | 988 | 991 | 993 | 994 |
| 3.0 | 0 | 1 | 3 | 6 | 12 | 21 | 34 | 53 | 78 | 113 | 156 | 210 | 272 | 342 | 418 | 498 | 577 | 653 | 723 | 786 | 839 | 883 | 917 | 943 | 962 | 975 | 983 | 989 | 992 | 994 | 995 |

## APPENDIX C

## PROGRAM USER'S GUIDE

This appendix presents the necessary instructions for using the computer simulation program that was developed during the research effort. The program can be used to simulate 100 paving days of Marshall test results to investigate the performance of different methods for determining the payment to be received for the lot of material. The program uses the current FAA Eastern Region payment schedule to determine the payment factor for the estimated PWL values. Although the Marshall properties were specifically addressed in this study, the program can be used for any correlated trivariate case by inputting the appropriate acceptance limits.

The program determines a composite Marshall payment factor using 7 different methods, including:

1. triple numerical integration to determine the overall PWL
2. multiplying the individual property PWL values
3. averaging the individual property PWL values
4. using the smallest individual property PWL value
5. multiplying the individual property payment factors
6. averaging the individual property payment factors
7. using the smallest individual property payment factor.

### Computer System Information

The program was written in FORTRAN and has been successfully executed using the FORTRAN77 and WATFIV compilers and an IBM 3081-K main-frame computer. The program should run on any system supporting the FORTRAN77 language. The program is supplied on computer tape and a complete program listing is also presented in Appendix D. The program is entitled ACCEPT, and it appears under this data set name on the tape. The necessary information to identify the tape and the program are listed below.

| | | |
|---|---|---|
| Volume Serial Number: | BURATI | |
| Label: | IBM standard label | |
| Density: | 6250 bpi | |
| Tracks: | 9 tracks | |
| Contents of Tape: | File Number | Data Set Name |
| | 1 | ACCEPT |

## Data Input Requirements

The required input data for the ACCEPT program include:

1. the acceptance limits for the properties being considered,
2. the means for the properties being considered and
3. the variance-covariance matrix for the properties being considered.

### Acceptance Limits

The program can be used to simulate any trivariate acceptance situation. All that is necessary is that the upper and lower acceptance limits for the 3 properties being considered be entered as program input. For the Marshall properties, the limits used in the simulations in the current research, the acceptance limits were:

$$1800 < \text{stability} < 8000$$

$$8 < \text{flow} < 16$$

$$2 < \text{air voids} < 5.$$

The program is designed for both upper and lower acceptance limits. In the case of a property with no upper limit, such as stability, it is necessary to input an artificial upper limit. This limit can be arbitrarily selected to insure it is so high that none of the simulated values will exceed it.

### Mean Values

The next data input to the program are the mean values for the 3 properties being simulated. These are the mean values for the population from which the simulated Marshall test results are drawn. In the research, the values from the 15 runway paving projects were used as the 'population statistics' in the various simulation analyses.

### Variance-Covariance Matrix

The final input data to the program are the values for the variance-covariance matrix for the 3 properties being simulated. The variance for each property can be calculated as the square of the standard deviation for the property. The covariance for each pair of properties can be calculated from the standard deviations for the properties and the correlation coefficient between the properties. Correlation coefficients can be estimated from those presented in this report, or can be calculated from historical data using the formula presented in Chapter II. Many hand-held calculators also have built-in routines for calculating the correlation coefficient between 2 sets of variables. A numerical example will help to illustrate the process for calculating covariances.

Suppose the following is known about a population of Marshall properties to be simulated:

STDs = stability standard deviation = 243.8

STDf = flow standard deviation = 0.84

STDv = air voids standard deviation = 0.62

C,sf = correlation between stability and flow = +0.30

C,sv = correlation between stability and air voids = -0.40

C,fv = correlation between flow and air voids = -0.40.

These are the pooled standard deviation values from the 1981 paving projects and the trend correlation values developed from all the projects considered in the research.

The variance for each of the properties can be calculated as follows:

VARs = stability variance = $(\text{STDs})^2 = (243.8)^2 = 59{,}438$

VARf = flow variance = $(\text{STDf})^2 = (0.84)^2 = 0.706$

VARv = air voids variance = $(\text{STDv})^2 = (0.62)^2 = 0.384$.

The covariance values can be calculated from the standard deviation and correlation coefficient values by the following relationship:

if,

COV,xy = covariance between properties X and Y

C,xy = correlation coefficient between properties X and Y

STDx = standard deviation of property X

STDy = standard deviation of property Y

then,

COV,xy = (C,xy) * (STDx) * (STDy).

Therefore, for the example being considered, the covariances are:

COV,sf = (C,sf) * (STDs) * (STDf)
= (0.30) * (243.8) * (0.84) = 61.438

COV,sv = (C,sv) * (STDs) * (STDv)
= (-0.40) * (243.8) * (0.62) = -60.462

COV,fv = (C,fv) * (STDf) * (STDv)
= (-0.40) * (0.84) * (0.62) = - 0.208.

The variance-covariance matrix can be represented as:

| | stability | flow | air voids |
|---|---|---|---|
| stability | 59438. | 61.438 | -60.462 |
| flow | 61.438 | 0.706 | - 0.208 |
| air voids | -60.462 | - 0.208 | 0.384 |

## Input Cards or Lines

All input values in the program are in the free format mode and are real vriables (i.e., they require a decimal point). A summary of the input values is presented below:

| Line | Variables |
|---|---|
| 1 | lower limit and upper limit for stability |
| 2 | lower limit and upper limit for flow |
| 3 | lower limit and upper limit for air voids |
| 4 | mean stability, mean flow and mean air voids |
| 5 | stability variance, stability-flow covariance and stability-air voids covariance |
| 6 | stability-flow covariance, flow variance and flow-air voids covariance |
| 7 | stability-air voids covariance, flow-air voids covariance and air voids variance. |

Using the notation presented above, and defining the stability, flow and air voids means by XBARs, XBARf and XBARv, respectively, and the upper and lower limits for stability, flow and air voids as ULs, LLs, ULf, LLf, ULv and LLv, respectively, the data input can be summarized as:

| Line | Variables | Example Problem Values |
|---|---|---|
| 1 | LLs ULs | 1800. 8000. |
| 2 | LLf ULf | 8. 16. |
| 3 | LLv ULv | 2. 5. |
| 4 | XBARs XBARv XBARv | 2671.8 10.55 3.47 |
| 5 | VARs COV,sf COV,sv | 59438. 61.438 -60.462 |
| 6 | COV,sf VARf COV,fv | 61.438 0.706 -0.208 |
| 7 | COV,sv COV,fv VARVv | -60.462 -0.208 0.384 |

## Program Output

The output from the program consists of the input data, the population PWL and payment factor, and the average values from the projects simulated. The program simulates 7 different payment determination methods. The 7 methods are identified on the program output, and include:

1. triple numerical integration
2. multiplying the individual PWL values
3. averaging the individual PWL values
4. using the smallest individual PWL value
5. multiplying the individual payment factors
6. averaging the individual payment factors
7. using the smallest individual payment factor.

The output for the program first prints an echo of the input data. The program then reports the PWL value for the population along with the payment factor based on the population PWL value. In addition, the following average values for the 100 paving days are reported for each of the 7 payment determination methods:

1. the mean of the 100 payment factors
2. the standard deviation of the 100 payment factors
3. the variance of the 100 payment factors
4. the bias
5. the mean square error (MSE) of the payment factors.

A sample output for program ACCEPT for the data from the example project in this appendix is presented in Exhibit C-1. The first 3 lines are the lower and upper acceptance limits for stability, flow and air voids, respectively. The next line presents the population mean values for stability, flow and air voids. Next, the variance-covariance matrix is printed. This allows the user to verify that the correct values were input to the program. The program then prints the population PWL

value and the corresponding payment factor. Finally, the simulation results are reported.

## Modifying the Payment Schedule

The program uses the current FAA Eastern Region density payment schedule for determining the payment factor for the PWL values that are estimated. If the user wishes to evaluate the performance of different price adjustment (i.e., payment ) schedules, then the program statements corresponding to the payment factor determination must be modified to reflect the different payment schedule. The statements that must be modified to use a different payment schedule can be identified in the program listing in Appendix D.

In Appendix D, the first column of numbers are the statement numbers for the program. The column of numbers on the right side of the page are the individual line numbers assigned to each line of the program. The payment factors are calculated at 3 different locations in the program. These locations are: statements 22 - 25, statements 40 - 43, and statements 46 - 49.

In statements 22 - 25, ACT is the population PWL value, and PAYFAC is the payment factor calculated using the population PWL value. In statements 40 - 43, P1(I) for I = 1,2,3 are the composite PWL values determined by multiplying, averaging, and using the smallest of the individual PWL values; and P2(I) for I = 1,2,3 are the corresponding composite payment factors. In statements 46 - 49, PWL(I) for I = 1,2,3 are the individual PWL values for stability, flow and air voids; and P(I) for I = 1,2,3 are the corresponding individual property payment factors. PWL(I) for I = 4 in statements 46 - 49 represents the PWL calculated by triple integration; and P(I) for I = 4 is the corresponding payment factor.

An example will help to illustrate how the program can be modified to use a different payment schedule. Suppose the following payment schedule is being considered:

| Estimated PWL | Payment Factor |
|---|---|
| 50 - 100 | pay factor = estimated PWL |
| Below 50 | 50. |

If this payment schedule were to be evaluated, the statements 22 - 25 would be removed and replaced with the following statements:

```
IF(ACT .GE. 50) PAYFAC = ACT
IF(ACT .LT. 50) PAYFAC = 50.
```

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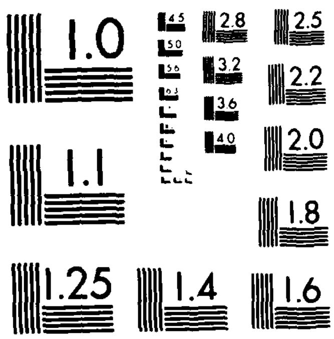
END
FILMED
DTIC




MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

and, statements 40 - 43 would be removed and replaced with the following statements:

```
IF(P1(I) .GE. 50) P2(I) = P1(I)
IF(P1(I) .LT. 50) P2(I) = 50.
```

and, statements 46 - 49 would be removed and replaced with the following statements:

```
IF(PWL(I) .GE. 50) P(I) = PWL(I)
IF(PWL(I) .LT. 50) P(I) = 50.
```

```
STAB LIMITS =  1800.0 AND 8000.0
FLOW LIMITS =    8.00 AND  16.00
VOIDS LIMITS =   2.00 AND   5.00


MEANS FOR STAB, FLOW, AND VOIDS =    2672.00     10.55     3.47


VARIANCE-COVARIANCE MATRIX

 59438.4414      61.4376     -60.4624
    61.4376       0.7056      -0.2083
   -60.4624      -0.2083       0.3844


POP PWL =  96.96   POP PAY FACTOR = 100.00
```

| METHOD | MEAN | STD DEV | VARIANCE | BIAS | MSE |
|---|---|---|---|---|---|
| 1 | 79.9 | 21.2 | 449.7 | -20.1 | 852.8 |
| 2 | 99.9 | 0.5 | 0.3 | -0.1 | 0.3 |
| 3 | 100.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 4 | 99.9 | 0.5 | 0.3 | -0.1 | 0.3 |
| 5 | 99.9 | 0.5 | 0.3 | -0.1 | 0.3 |
| 6 | 100.0 | 0.2 | 0.0 | -0.0 | 0.0 |
| 7 | 99.9 | 0.5 | 0.3 | -0.1 | 0.3 |

METHOD
1. TRIPLE NUMERICAL INTEGRATION
2. MULTIPLYING THE INDIVIDUAL PWL VALUES
3. AVERAGING THE INDIVIDUAL PWL VALUES
4. USING THE SMALLEST INDIVIDUAL PWL VALUE
5. MULTIPLYING THE INDIVIDUAL PAYMENT FACTORS
6. AVERAGING THE INDIVIDUAL PAYMENT FACTORS
7. USING THE SMALLEST INDIVIDUAL PAYMENT FACTOR

EXHIBIT C-1. SIMULATION PROGRAM OUTPUT FOR THE EXAMPLE PROBLEM

# APPENDIX D

## SIMULATION PROGRAM LISTING

```
      DIMENSION CV(3,3),B(3,3),XBAR(7),COV(7,7),AVG(3),STD(3),R(3,3),
     1PAY(100,7),AV(3),U(20),V(20),PWL(7),P(7), BIAS(7),X(20,3),P1(3),
     1P2(3)
      REAL MSE(7)
      DO 10 I=1,3
      READ 88,U(I),V(I)
   88 FORMAT(2F10.0)
   10 CONTINUE
      PRINT 40, U(1),V(1),U(2),V(2),U(3),V(3)
   40 FORMAT(//' STAB LIMITS =  ',F6.1,' AND ',F6.1,/,' FLOW LIMITS = ',
     +F6.2,' AND ',F6.2,/,' VOIDS LIMITS = ',F6.2,' AND ',F6.2)
      READ  89,(AV(I),I=1,3)
   89 FORMAT(3F10.0)
      PRINT 50,(AV(I),I=1,3)
   50 FORMAT(//' MEANS FOR STAB, FLOW, AND VOIDS = ',3F10.2)
      PRINT 60
   60 FORMAT(//' VARIANCE-COVARIANCE MATRIX'/)
      DO 15 I=1,3
      READ  86,(CV(I,J),J=1,3)
   86 FORMAT(3F10.0)
   15 PRINT 70,(CV(I,J),J=1,3)
      CALL CAL(AV,CV,U,V,ACT)
      ACT=ACT*100.
      PAYFAC=0.
      IF(ACT .GE. 90) PAYFAC=100.
      IF(ACT .LT. 90 .AND. ACT .GE. 80) PAYFAC= .5*ACT+55.
      IF(ACT .LT. 80 .AND. ACT .GE. 65) PAYFAC= 2.*ACT-65.
      IF(ACT .LT. 65) PAYFAC=50.
      PRINT 80, ACT,PAYFAC
   80 FORMAT(//' POP PWL = ',F6.2,'  POP PAY FACTOR = ',F6.2)
   70 FORMAT(3F12.4)
      CALL SCDA(CV,B)
      NSAMPL = 4
      IZ=341547
      M=1
      DO 5 ITER = 1,250
      CALL SIM(AV,B,U,V,PWL,IZ,ERR,AVG,STD,R,NSAMPL,X)
      IF(ERR .EQ. 1) GO TO 5
      P1(1)=(PWL(1)*PWL(2)*PWL(3))/10000.
      P1(2)=(PWL(1)+PWL(2)+PWL(3))/3.
      P1(3)=(AMIN1(PWL(1),PWL(2),PWL(3)))
      DO 2 I=1,3
      IF(P1(I) .GE. 90 ) P2(I)=100.
      IF(P1(I) .LT. 90 .AND. P1(I) .GE. 80) P2(I)=.5*P1(I)+55
      IF(P1(I) .LT. 80 .AND. P1(I) .GE. 65) P2(I)=2.*P1(I)-65
      IF(P1(I) .LT. 65) P2(I) = 50.
    2 CONTINUE
      DO 1 I=1,4
      IF(PWL(I) .GE. 90 ) P(I)=100.
      IF(PWL(I) .LT. 90 .AND. PWL(I) .GE. 80) P(I)=.5*PWL(I)+55
      IF(PWL(I) .LT. 80 .AND. PWL(I) .GE. 65) P(I)=2.*PWL(I)-65
      IF(PWL(I) .LT. 65) P(I) = 50.
    1 CONTINUE
      PAY(M,2)=P2(1)
      PAY(M,3)=P2(2)
      PAY(M,4)=P2(3)
      PAY(M,5)=(P(1) * P(2) * P(3))/10000.
      PAY(M,6)=(P(1) + P(2) + P(3))/3.
```

(CONTINUE)

```
      PAY(M,7)=(AMIN1(P(1),P(2),P(3)))
      PAY(M,1)=P(4)
      IF(M .EQ.100) GO TO 22
      M=M+1
    5 CONTINUE
   22 CONTINUE
      CALL STAT(PAY,XBAR,COV,100,7)
      PRINT 90
   90 FORMAT(//'   METHOD      MEAN      STD DEV  VARIANCE     BIAS      M
     +SE    '/)
      DO 25 I=1,7
         BIAS(I)=XBAR(I)-  PAYFAC
      MSE(I)=COV(I,I) + BIAS(I) * BIAS(I)
   25 PRINT 30,I,XBAR(I),SQRT(COV(I,I)),COV(I,I),BIAS(I),MSE(I)
   30 FORMAT(I10,5F10.1)
      PRINT 64
   64 FORMAT(/3X,'METHOD'/7X,'1. TRIPLE NUMERICAL INTEGRATION'/7X,
     *'2. MULTIPLYING THE INDIVIDUAL PWL VALUES',/7X,
     *'3. AVERAGING THE INDIVIDUAL PWL VALUES',/7X,
     *'4. USING THE SMALLEST INDIVIDUAL PWL VALUE',/7X,
     *'5. MULTIPLYING THE INDIVIDUAL PAYMENT FACTORS',/7X,
     *'6. AVERAGING THE INDIVIDUAL PAYMENT FACTORS',/7X,
     *'7. USING THE SMALLEST INDIVIDUAL PAYMENT FACTOR')
      STOP
      END


      SUBROUTINE SIM(AV,B,USPEC,VSPEC,PWL,IZ,ERR,XBAR,STD,COR,NSAMPL,X)
C  **  GENERATE THE VECTOR OF MARSHALL VARIABLES FOUR TIMES
C  **  COMPUTE SAMPLE STATISTICS FROM THE FOUR SAMPLES
      DIMENSION AV(3),XBAR(3),Z(3),PWL(7),A(3),D(3),QL(3),QU(3),COR(3,3)
     1,STD(3),B(3,3),C(3,3),U(3),V(3),X(NSAMPL,03),USPEC(3),VSPEC(3),
     2R(3,3)
     NVAR=3
     DO 15 M=1,NSAMPL
     DO  5 I=1,NVAR
    5 Z(I)=RNOR(IZ)
      DO 10 I=1,NVAR
      X(M,I)=AV(I)
      DO 10 J=1,I
   10 X(M,I)=X(M,I)+B(I,J)*Z(J)
   15 CONTINUE
      CALL STAT(X,XBAR,C,NSAMPL,NVAR)
      DO 20 I=1,NVAR
   20 STD(I)=SQRT(C(I,I))
      DO 30 I=1,NVAR
      DO 25 J=1,NVAR
      R(I,J)=C(I,J)/(STD(I)*STD(J))
   25 COR(I,J)=R(I,J)
      QL(I)=(XBAR(I)-USPEC(I))/STD(I)
      QU(I)=(VSPEC(I)-XBAR(I))/STD(I)
      U(I)=(USPEC(I)-XBAR(I))/STD(I)
      V(I)=(VSPEC(I)-XBAR(I))/STD(I)
      IF(U(I) .LT. -3.5) U(I)=-3.5
      IF(V(I) .GT.  3.5) V(I)= 3.5
   30 CONTINUE
      CALL    SUM(U,V,ANS,R,NVAR,ERR)
      PWL(4) = ANS*100.
      DO 40 K=1,NVAR
      G=QL(K)
      H=QU(K)
      A(1)=U(K)
      D(1)=V(K)
   40 PWL(K) =PWLHAT(G,H)
      RETURN
      END
```

(CONTINUE)

```
      FUNCTION FUN(X,R,NVAR,ERR)
      DIMENSION R(3,3),L(3),M(3),X(3),Y(3,3),Z(3,3)
      IF(NVAR .GT. 1) GO TO 5
      FUN=(1./SQRT(6.2836))* EXP(-0.5*X(1)*X(1))
      GO TO 15
    5 DET= R(1,1)*(R(2,2)*R(3,3)-R(3,2)*R(2,3))
     1    -R(2,1)*(R(1,2)*R(3,3)-R(3,2)*R(1,3))
     2    +R(3,1)*(R(1,2)*R(2,3)-R(2,2)*R(1,3))
      ERR=1.
      IF(DET .LE. 0.) GO TO 15
      ERR=0.
      DS=SQRT(DET)
      CONST=1./(6.2836**1.5  * DS)
      CALL MINV(R,NVAR,F,L,M)
      CALL GMPRD(X,R,Y,1,3,3)
      CALL GMPRD(Y,X,Z,1,3,1)
      Z1=-0.5*Z(1,1)
      IF(Z1.LT.-160) GOTO 10
      FUN=CONST*(EXP(Z1))
      GOTO 15
   10 FUN=0.
   15 RETURN
      END



      FUNCTION RNOR(IR)
C   GENERATES A RANDOM NORMAL NUMBER
      DATA I/0/
      IF(I.GT.0) GOTO 30
   10 CALL RANDU(IR,IRR,U)
      IR=IRR
      X=2.0*U-1.0
      CALL RANDU(IR,IRR,U)
      IR=IRR
      Y=2.0*U-1.0
      S=X*X+Y*Y
      IF(S.GE.1.)GOTO 10
      S=SQRT(-2.0*ALOG(S)/S)
      RNOR=X*S
      GO2=Y*S
      I=1
      GOTO 40
   30 RNOR=GO2
      I=0
   40  RETURN
      END



      SUBROUTINE SCDA(C,R)
      DIMENSION C(3,3),R(3,3)
      R(1,2)=0.
      R(1,3)=0.
      R(2,3)=0.
      DO 15 K=1,3
      R(K,K)=SQRT(C(K,K))
      DO 10 J=1,3
      IF(J.LE.K) GOTO 10
      R(J,K)=C(J,K)/R(K,K)
      KK=K+1
      DO 5 I=KK,J
      C(J,I)=C(J,I)-C(I,K)*C(J,K)/C(K,K)
    5 CONTINUE
   10 CONTINUE
   15 CONTINUE
      RETURN
      END
```

(CONTINUE)

```
      SUBROUTINE GMPRD(A,B,R,N,M,L)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
      JI=J-N
      IB=IK
      R(IR)=0
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END
```

```
      SUBROUTINE STAT(X,XBAR,COV,NSAMPL,NVAR)
      DIMENSION X(NSAMPL,NVAR),XBAR(NVAR),COV(NVAR,NVAR)
C     COMPUTE SAMPLE MEAN OF VARIABLE I
      DO 45 I=1,NVAR
      XBAR(I)=0.
      DO 40 M=1,NSAMPL
   40 XBAR(I)=XBAR(I)+X(M,I)
   45 XBAR(I)=XBAR(I)/FLOAT(NSAMPL)
C     COMPUTE THE SAMPLE COVARIANCE
      DO 60 I=1,NVAR
      DO 55 J=1,NVAR
      SUM=0.
      DO 50 M=1,NSAMPL
   50 SUM=SUM+X(M,I)*X(M,J)
      SUM=SUM-NSAMPL*XBAR(I)*XBAR(J)
   55 COV(I,J)=SUM/FLOAT(NSAMPL-1)
   60 CONTINUE
      RETURN
      END
```

(CONTINUE)

```
      SUBROUTINE SUM(U,V,ANS,C,N,ERR)
      DIMENSION X(9,3),W(9,3),Z(3),M(3),N1(3),U(3),V(3),C(3,3),RANGE(3)
      DO 1 I=1,N
      N1(I)=9
      W(1,I)=1.
      W(2,I)=4.
      W(3,I)=2.
      W(4,I)=4.
      W(5,I)=2.
      W(6,I)=4.
      W(7,I)=2.
      W(8,I)=4.
      W(9,I)=1.
    1 CONTINUE
      WSUM=0.
      DO 15 I=1,N
      X(1,I)=U(I)
      RANGE(I)=V(I)-U(I)
      DO 15 J=2,9
      X(J,I)=X(J-1,I) + RANGE(I)/8.
   15 CONTINUE
      DO 10 I=1,N
   10 M(I)=1
      N2=N+1
      ANS=0.
    6 K=1
      W1=1.
      DO 2 I=1,N
      M1=M(I)
      Z(I)=X(M1,I)
    2 W1=W1*W(M1,I)
      WSUM=WSUM+W1
      ANS=ANS+W1*FUN(Z,C,N,ERR)
    8 IF(M(K)  -  N1(K)) 3,4,5
    5 STOP
    3 M(K)=M(K)+1
      GOTO 6
    4 M(K)=1
      K=K+1
      IF(K  -  N2) 8,7,5
    7 A=RANGE(1)
      IF(N .EQ. 1) GO TO 100
      DO 50 I=2,N
   50 A=A*RANGE(I)
  100 ANS=A*ANS/WSUM
      RETURN
      END
```

(CONTINUE)

```
      FUNCTION PWLHAT (QL,QU)
C
C-----------------------------------------------------------------
C
C   SUBROUTINE PWLEST ESTIMATES THE PWL VALUE
C   USING THE QUALITY INDEX APPROACH
C
C-----------------------------------------------------------------
C
      IMPLICIT REAL*4 (M)
      DIMENSION PWL(101),Z(101)
      DATA PWL/101*0.0/
      DATA Z/1.5000,1.4700,1.4400,1.4100,1.3800,1.3500,
     +       1.3200,1.2900,1.2600,1.2300,1.2000,1.1700,
     +       1.1400,1.1100,1.0800,1.0500,1.0200,0.9900,
     +       0.9600,0.9300,0.9000,0.8700,0.8400,0.8100,
     +       0.7800,0.7500,0.7200,0.6900,0.6600,0.6300,
     +       0.6000,0.5700,0.5400,0.5100,0.4800,0.4500,
     +       0.4200,0.3900,0.3600,0.3300,0.3000,0.2700,
     +       0.2400,0.2100,0.1800,0.1500,0.1200,0.0900,
     +       0.0600,0.0300,0.0000,-0.0300,-0.0600,-0.0900,
     +      -0.1200,-0.1500,-0.1800,-0.2100,-0.2400,
     +      -0.2700,-0.3000,-0.3300,-0.3600,-0.3900,
     +      -0.4200,-0.4500,-0.4800,-0.5100,-0.5400,
     +      -0.5700,-0.6000,-0.6300,-0.6600,-0.6900,
     +      -0.7200,-0.7500,-0.7800,-0.8100,-0.8400,
     +      -0.8700,-0.9000,-0.9300,-0.9600,-0.9900,
     +      -1.0200,-1.0500,-1.0800,-1.1100,-1.1400,
     +      -1.1700,-1.2000,-1.2300,-1.2600,-1.2900,
     +      -1.3200,-1.3500,-1.3800,-1.4100,-1.4400,
     +       -1.4700,-1.5000/
      DO 19 I=1,100
      PWL(101-I)=Z(I)
   19 CONTINUE
      IF(QL.LT.PWL(100)) GO TO 10
      PL = 0.000
      GO TO 40
   10 DO 20 I = 1,99
        JJ = 100 - I
        AZZZ = PWL(JJ)
        IF(QL.GT.AZZZ) GO TO 30
   20 CONTINUE
      PL = 100.000
      PU = 0.000
      GO TO 80
   30 F = (QL-PWL(JJ))/(PWL(JJ+1)-PWL(JJ))
      PL = 100.000-JJ-F
   40 IF(QU.LT.PWL(100)) GO TO 50
      PU = 0.000
      GO TO 80
   50 DO 60 I = 1,99
        JJ = 100-I
        AZZZ = PWL(JJ)
        IF(QU.GT.AZZZ) GO TO 70
   60 CONTINUE
      PU = 100.000
      GO TO 80
   70 F = (QU-PWL(JJ))/(PWL(JJ+1)-PWL(JJ))
      PU = 100.000-JJ-F
   80 PD = (100.000-PU-PL)
      PWLHAT = PD
      RETURN
      END
```

(CONTINUE)

```
      SUBROUTINE CAL(AV,COV,USPEC,VSPEC,ACT)
      DIMENSION AV(3),CORR(3,3),
     1STD(3),COV(3,3),U(3),V(3),USPEC(3),VSPEC(3)
      NSAMPL=4
      NVAR=3
      DO 20 I=1,NVAR
   20 STD(I)=SQRT(COV(I,I))
      DO 30 I=1,NVAR
      DO 25 J=1,NVAR
   25 CORR(I,J)=COV(I,J)/(STD(I)*STD(J))
      U(I)=(USPEC(I)-AV(I))/STD(I)
      V(I)=(VSPEC(I)-AV(I))/STD(I)
      IF(U(I) .LT. -3.5) U(I)=-3.5
      IF(V(I) .GT.  3.5) V(I)= 3.5
   30 CONTINUE
      CALL     SUM(U,V,ANS,CORR,NVAR,ERR)
      ACT = ANS
      RETURN
      END
```

# END

# FILMED

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# DTIC